A METHODOLOGY FOR VEHICLE TRAJECTORY-BASED CALIBRATION OF MICROSIMULATION MODELS

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Introduction

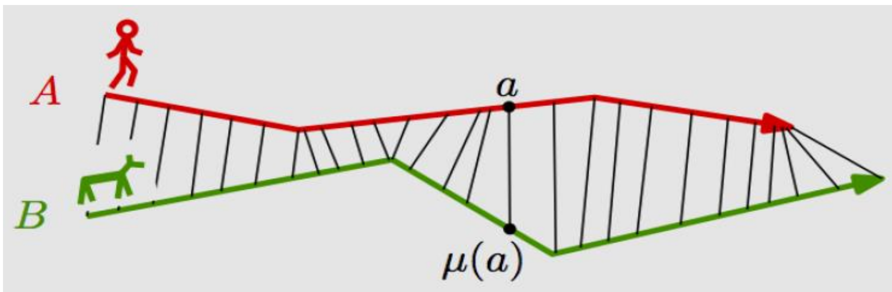
Traffic modelers have long been concerned about the validity of applying microsimulation models to forecast future conditions, even when present-day performance measures are closely replicated. This is because vehicle trajectories often reveal starkly different car-following and lane-changing behaviors between simulated drivers and real-world drivers. Such difference may compromise the use of simulation results in understanding microscopic phenomena (e.g., queue propagation, lane changes, safety) and evaluating measures to improve microscopic traffic performance. Fortunately, new data collection technologies and research findings are lowering the costs and risks of trajectory-based calibration. This presentation describes a proposed trajectory-based calibration methodology developed as part of a Federal Highway Administration (FHWA) research project. The methodology was designed to achieve the right balance between practicality for State agencies, robustness of calibration accuracy, and flexibility to accommodate project-specific needs.

The team reviewed literature in the major categories of trajectory-level data collection, trajectory data formats, data processing, common trajectory data errors, and applications of trajectory data in the calibration of microsimulation models. Key takeaways included:

- The team should document data categories when developing the data collection plan, to help define limitations of the proposed calibration methodology. (Daamen et al. 2014)
- To remove noises in trajectory, many methods have been proposed to smooth trajectories including moving average algorithms (Duret et al. 2008, Hamdar and Mahmassani 2008, Thiemann et al. 2008), smoothing algorithms (Punzo et al. 2005, Lu and Skabardonis 2007, Toledo et al. 2007), and Kalman filtering (Ervin et al. 1991, Ma and Andréasson 2005, Punzo et al. 2005). Customized algorithms are also proposed to check the consistency, feasibility, and safety of trajectories.
- GPS based trajectory data may have relatively large location errors, but these can be corrected with RTK (real-time kinematics) techniques to get centimeter level accuracy. (Lee and Jee 2010)
- Punzo et al. (2015) provided a methodology to reduce the number of parameters to calibrate without degrading the calibration robustness. A Monte Carlo framework was adopted in the paper to calculate the sensitivity. Distances between the observed and simulated trajectories were calculated in terms of root-mean-square error (RMSE) of the instantaneous speed or spacing. The process is iterated until the number of evaluations is sufficient for the calculated indices to be stable.

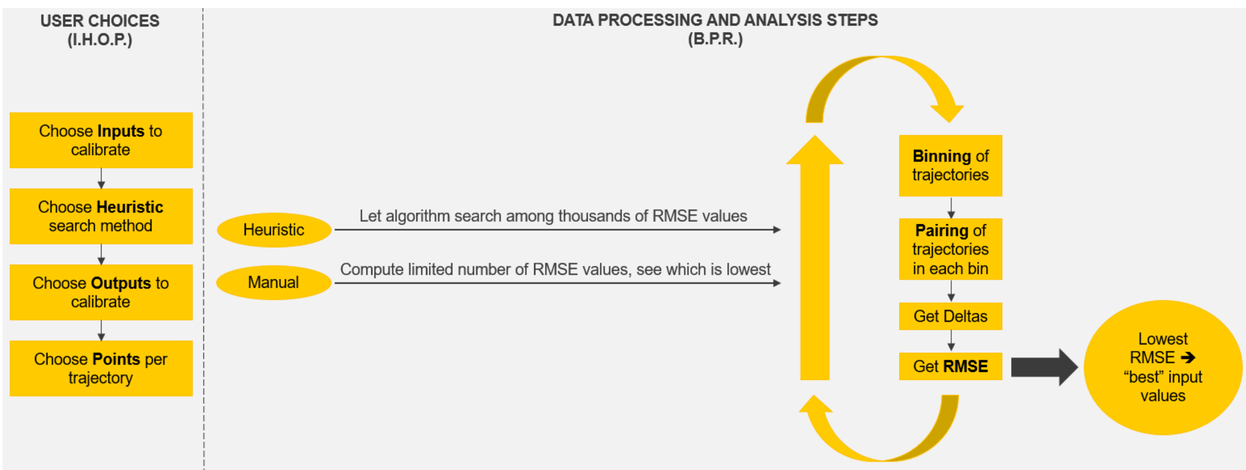
Methodology

The calibration methodology was developed slowly, deliberately, and organically, from the ground up. The project scope required a methodology that would explicitly consider full-set trajectories (Figure 1), as opposed to surrogate measures or pseudo-trajectory measures. A preliminary calibration of demand volumes, to achieve more accurate segment throughput, was suggested as a pre-requisite to the subsequent trajectory-based calibration of driver behaviors. A seven-step trajectory-based calibration methodology (Figure 2) was conceived. Some of these seven steps (e.g., choosing which input parameters to calibrate) were relatively non-technical, and would probably be required for any calibration methodology. Similarly, the well-known simulation-based optimization framework would support both common heuristic search methods, as well as semi-exhaustive enumeration methods (also known as directed brute force).



Source: FHWA

Figure 1. Diagram. Comparison of Full-Set Trajectories



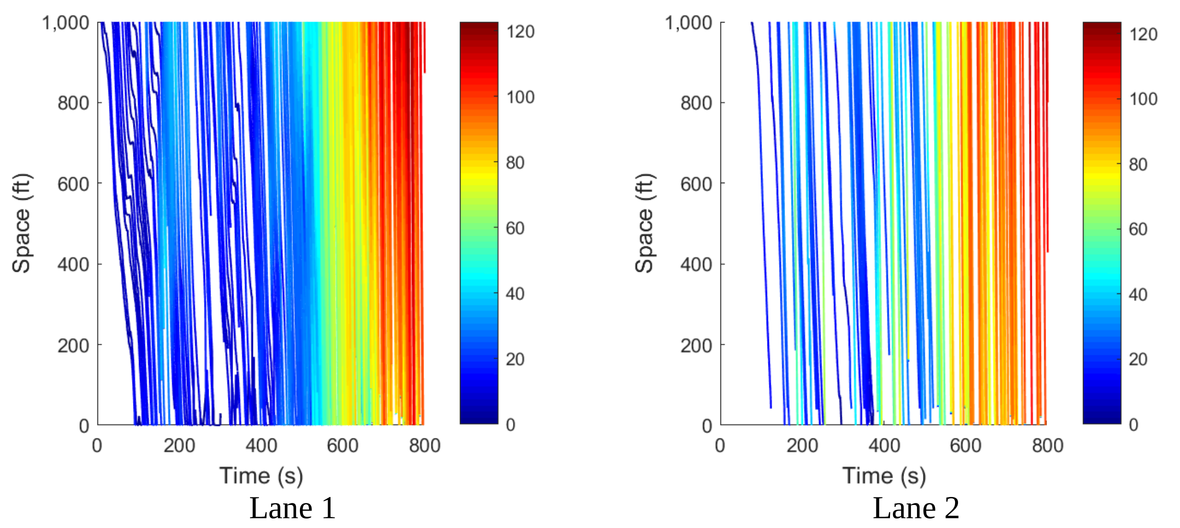
Source: FHWA

Figure 2. Flowchart. Proposed 7-Step Trajectory-Based Calibration Methodology

The most sensitive methodological elements eventually involved steps 5 and 6: the binning and pairing of trajectories for subsequent comparison, respectively. The binning step would separate simulated and field-measured trajectories into groups of similar characteristics (e.g., vehicle type, driver type, origin, destination, weather), to facilitate apples-to-apples comparison. A very small number of vehicle types and driver types was recommended, to provide a balance between practicality and robustness. Although each upstream lane number was recommended as a unique origin, all downstream mainline lanes were grouped into a single destination, for better analysis of lane-changing behaviors. Finally, weather-related bins would only be feasible if the agency has both access to poor weather trajectory data, plus the ability to conduct robust microsimulation of poor weather conditions. Regarding the pairing of trajectories following binning, a purely timestamp-centric approach was suggested, to circumvent inevitable biases in the congestion levels.

Results

To demonstrate and validate the proposed microsimulation procedure, extensive field data collection efforts were conducted at several sites across the country (i.e., I-95 in Virginia, I-270 in Maryland, I-75 in Florida, and I-15 in California). On I-95, I-15 and I-270, video data were collected by drones. At each site, three or four locations were selected to capture traffic conditions for about one hour during rush hours on weekdays. On I-75, traffic on an 8,000-foot-long road segment was captured by three 8K cameras on a helicopter for two hours. The video data were then processed, and vehicle trajectories were extracted by the Video-based Intelligent Road Traffic Universal Analysis Tool (VIRTUAL) developed at the University of South Florida. The extracted trajectory data format was kept consistent with Next Generation Simulation (NGSIM) data for the convenience of potential users. Due to the quality of the videos, a vehicle might not be always identified at the correct position in every frame. To mitigate this issue and improve the output trajectories, algorithms were developed to enhance vehicle detection and connect broken trajectories. Further, a data collection algorithm including a low-pass filter was used to ensure the extracted trajectories satisfied kinematic and safety constraints. The extracted trajectories were shown to have consistent and reasonable locations, speeds, and accelerations. Figure 3 below plots sampled trajectories from the I-95 Virginia network.



Source: FHWA

Figure 3. Line Graph. Sampled Trajectories on the I-95 Virginia Network

Two different microsimulation platforms were used to demonstrate and validate the proposed procedure with collected data. Experimental outcomes confirmed some of the worst fears that microsimulation experts have long held: if trajectories are not explicitly considered during the calibration process, the resulting car-following and lane-changing behaviors may be grossly inaccurate, even if the aggregate performance measures are "spot on". Figure 4 provides an early sample of such results, in which the trajectory RMSE results reveal much less accuracy following a purely traditional calibration (i.e., denoted as "weight = 0" below).

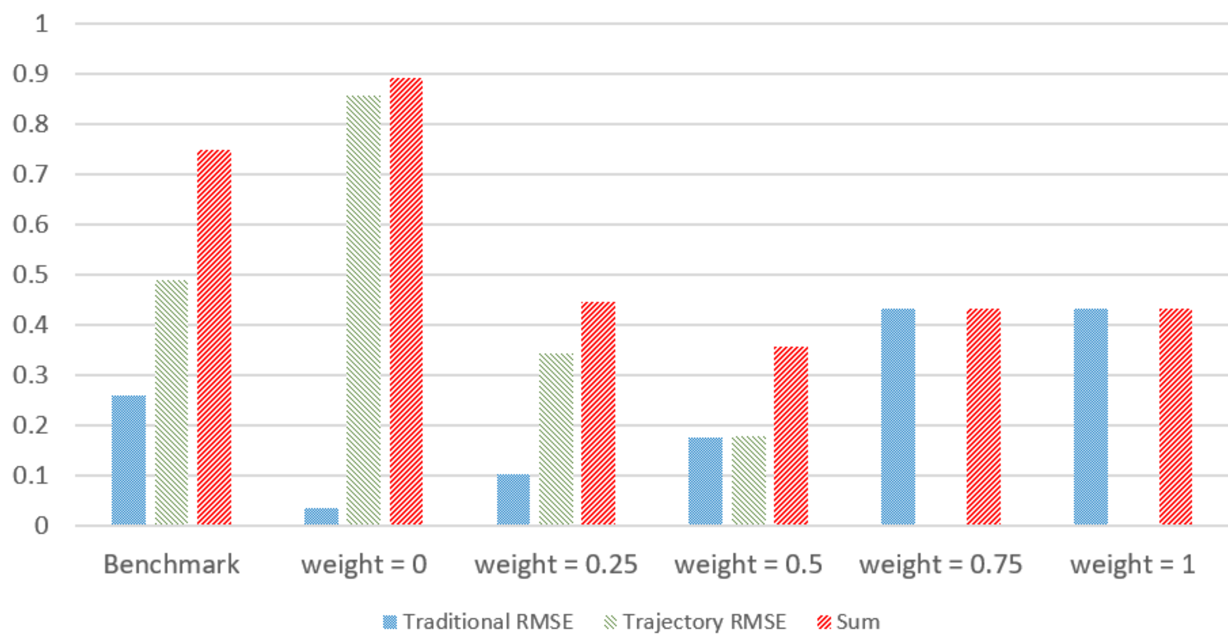


Figure 4. Bar Chart. Preliminary Calibration Results for the I-95 Virginia Network

Conclusion

Based on the initial experimental results, the need for practice-ready, trajectory-based calibration tools is more critical than ever. On the positive side, trajectory-based calibration is now much closer to being practice-ready; due to improved data collection technologies, improved data processing technologies, and the proposed calibration methodology. However, in order for the process to be truly practice-ready, more software development, automation, integration, and interoperability are needed at all three of these levels (i.e., trajectory data collection, trajectory data processing, trajectory calibration calculations). Moreover, the team's initial research and development results begged many new questions to potentially be examined by future research.

For example, if the vehicle trajectories and traditional aggregate measures are both considered "ground truth", how much emphasis or weighting should agencies place on both forms of output? Would this weighting be governed by congestion regime (i.e., below capacity, near capacity, at capacity, above capacity), evolution of data collection technologies, or both? Will trajectory data ever become reliable enough to safely and fully exclude traditional measures from the process? Further, how much emphasis or weighting should agencies place on calibration of car-following relative to lane-changing? Would this weighting be governed by congestion regime, structure of the driver behavior model, or both? What level of aggregation (or binning) from sub-second detailed trajectory data to minute-level aggregated traffic counts is the most appropriate for calibrating simulation models? Finally, what research and development efforts would best streamline the potential "pain points" of aerial data collection, automated conversion of simulation-plus-field trajectory data to NGSIM-type format, automated binning/pairing of the post-processed trajectories, and automated calculation of objective function values (e.g., root mean square error)?

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