Traffic Light Control Strategy Based on Macroscopic Fundamental Diagram

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Introduction

Daganzo and Geroliminis proposed the concept of Macroscopic Fundamental diagram, and used empirical data to show the relationship between total road demand and capacity in urban networks[1]. When studying the impact of turning traffic flow on the Macroscopic Fundamental diagram, Geroliminis et al. found that the existence of left-turning traffic flow at the intersection affected the maximum value of the traffic flow[2].

When assigning the demand on the traffic network, the OD matrix is necessary. However, as the fast expansion of Mobility as a Service (MaaS) companies such as Uber and Lyft, the OD matrix is difficult to reach as the MaaS companies generally keep these information as a secret to remain the competitive advantage over their rivals. To overcome this limitation, we provide a method to divide different trip based on check whether the vehicle is detour. The turn probability of each vehicle is predetermined to fit the observable data of real world. We track the vehicle trace and check whether they are in the shortest path. If observed the vehicle can reach the current position from a shorter path, we divide the trip as a different one and reset the trace of the vehicle.

Combined with the state transition matrix, we firstly makes probability density of the different travel distance expressed as a function of the turn probability and the road length so that the travel distance distribution can be turned into the turning probability of residents in front of the intersection. And then a multi-lane urban road network of 8x8 size model based on Nasch model[3] is constructed to analyzes the influence of different factors on the Macroscopic Fundamental diagram under different turning probability combinations.

Methodology

The travel distance in the model is divided according to the shortest path principle. State (n) indicates the state of the vehicle after making a left, straight, or right turn at the *n*th intersection. The state (n) is any one of five integers (-2, -1, 0, 1, 2), where state (n) =-2 means to end the trip with a left turn, and state (n) = 2 means to turn right end of the trip. So the state transition diagram can be drawn as figure 1, the a is the probability of the vehicle turning left and the b is the probability of the vehicle turning right.

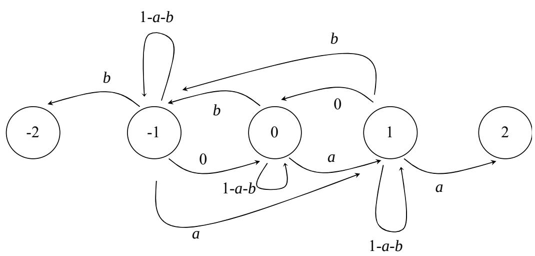


Figure 1 the state transition diagram

Assuming the system initial state matrix is $S(0) = (0,0,1,0,0)$, then the system state after n intersections is expressed as follows, and s_1 s_2 s_3 s_4 s_5 represent the element of $S(n)$,

$$S(n) = (s_1, s_2, s_3, s_4, s_5) = (0,0,1,0,0) \times A^n$$

Then the probability density of the travel distance $f(n)$ can be expressed as $\{f(n) = s_1 + s_5, \forall \mathrm{n} \geq 2\}$,which can be solved as follows

$$f(n) = s_1 + s_5 = \frac{(a+b)(1-b-a)^n}{a+b-1} - \frac{b\left(b+(ab)^{\frac{1}{2}}\right)\left((ab)^{\frac{1}{2}}-b-a+1\right)^n\left(a+b+(ab)^{\frac{1}{2}}-1\right)}{2(ab)^{\frac{1}{2}}(a^2+ab-2a+b^2-2b+1)}$$

$$-\frac{\left(b+(ab)^{\frac{1}{2}}\right)\left((ab)^{\frac{1}{2}}-b-a+1\right)^n\left(a^2b-a(ab)^{\frac{1}{2}}+(ab)^{\frac{3}{2}}+a^2(ab)^{\frac{1}{2}}\right)}{2b(ab)^{\frac{1}{2}}(a^2+ab-2a+b^2-2b+1)}$$

$$+\frac{b\left(b-(ab)^{\frac{1}{2}}\right)\left(a+b-(ab)^{\frac{1}{2}}-1\right)\left(1-b-(ab)^{\frac{1}{2}}-a\right)^n}{2(ab)^{\frac{1}{2}}(a^2+ab-2a+b^2-2b+1)} - \frac{a\left(b-(ab)^{\frac{1}{2}}\right)\left(a+b-(ab)^{\frac{1}{2}}-1\right)\left(1-b-(ab)^{\frac{1}{2}}-a\right)^n}{2b(a^2+ab-2a+b^2-2b+1)}$$

It means that when the travel distance distribution is known, the turning probability at the intersection can be solved.

Simulation Results

Since we can deduce the turn probability, the next step is to find the best traffic light timing plans by using macroscopic fundamental diagram as an evaluation index .The simulation model is based on cellular automata shown in Figure 1. Each arterial road is divided into two straight lanes and an extra left-turn lane. The rightmost lane can be used to turn right or go straight at the intersection, and the vehicles in the middle lane can only choose to go straight. vehicles in the left-turn lane can also only make left-turns on the intersection. And P_l P_s P_r is the probability of turning left, turning right, and going straight respectively. The update rules of vehicles follow the classic Nasch model and the update rules of traffic light follow the classic four phases model. Vehicles emerge from the border and enter the system and the probability of occurrence obeys pulse distribution.

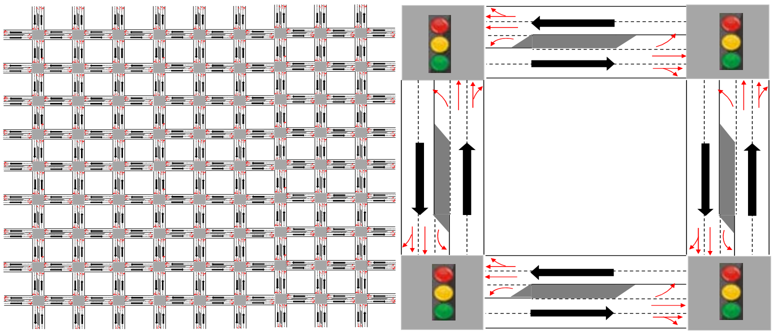
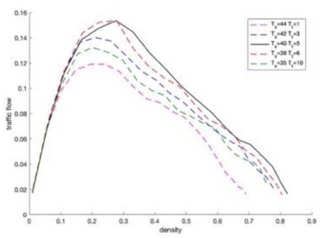
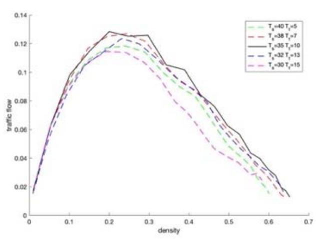


Figure 2 the model diagra

According to the simulation, we finds that when the traffic light cycle is fixed, the timing plan having the straight green ratios equal to the sum of the straight forward probability and the left turn probability can result in the highest curve of macroscopic fundamental diagram, which can be seen in figure 5. At the same time, under the optimal timing set, the shorter the length of the traffic light cycle, the higher macroscopic fundamental diagram curve will be, which can be seen in figure 6.

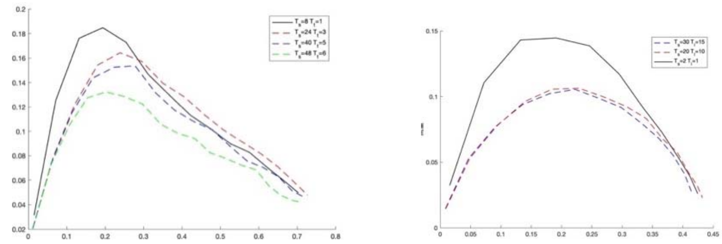


(a) $P_l = 0.1$ $P_s = 0.8$ $P_r = 0.1$



(b) $P_l = 0.2$ $P_s = 0.7$ $P_r = 0.1$

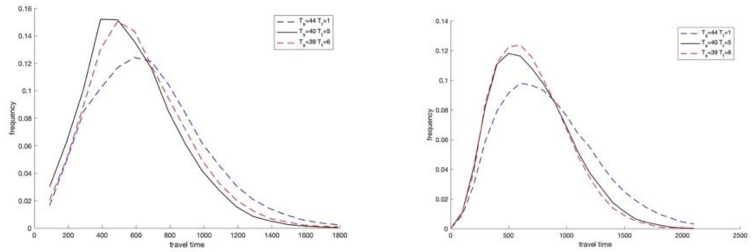
Figure 3 the simulation under fixed traffic light cycle



(a) $P_l = 0.1$ $P_s = 0.8$ $P_r = 0.1$ (b) $P_l = 0.3$ $P_s = 0.6$ $P_r = 0.1$

Figure 4 the influence of the length of traffic light cycle

The optimal timing set simultaneously has shorter travel time, indicating the validity of the macroscopic fundamental diagram as an evaluation index, which can be seen in figure 7.



(a) $P_l = 0.1$ $P_s = 0.8$ $P_r = 0.1$ (b) $P_l = 0.1$ $P_s = 0.7$ $P_r = 0.2$

Figure 5 the travel time under different traffic light timing plans

Conclusion

when the straight green ratios is equal to the sum of the straight forward probability and the left turn probability, the urban road has the highest macroscopic fundamental diagram curve under the same traffic light cycle length, and any combination of the traffic lights that increase or decrease the straight spilt will lead to the decline of the curve. When the optimal green ratio is fixed, the shorter the length of the traffic light cycle, the higher macroscopic fundamental diagram curve will be obtained. At the same time, under the optimal green ratio set, the travel time of residents is obviously shorter. The corresponding turn probability can be solved under the known travel distribution and road length, which helps to fit the optimal green ratio.

Reference

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