Traffic Assignment Analysis of Traffic Networks with Max-pressure Control

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1 Introduction

Intersection control is a widely used way to improve the efficiency and the safety of traffic networks. As an intersection control algorithm, max-pressure control was shown to have smaller queue length or delays than fixed-time signal controls (2) and fully adaptive traffic signal controls (12, 5, 8) in simulations. Some studies proved its network-level property of stabilizing the total queue length in different assumptions, such as infinite queue capacity (11), finite queue capacity (14), and constant cycle length (1). However, when designing the max-pressure control policy or testing its performance, most studies assumed constant average turning proportions from one link to another, which indicates fixed route choice behaviors of road users. This assumption may not be realistic when road users react to the intersection control by switching to paths with smaller link travels and intersection delays.

In existing literature that considered both route choice and intersection control, some studies solved the combined traffic assignment and intersection control problem on networks with traditional intersection controls. For example, some studies created bi-level optimization models and solved them by iterative algorithms (4, 9). Li et al. (3) established the model as a space-phase-time hyper-network and used a Lagrangian-relaxation-based optimization to decouple the model. Other studies formulated the model as an integrated global optimization problem (4, 9, 13, 3). Ukkusuri et al. (10) modeled the combined dynamic user equilibrium and signal control problem as a Nash-Cournot game and a Stackelberg game and solved the problem using a heuristic algorithm.

There is one study that considering the effect of route choice in the design of the max-pressure control. Le et al. (1) proposed a max-pressure control policy aimed to minimize the traffic inequality in the network, represented by a Gini index function. Another study from Smith (6) used a pressure-based policy P_0 to compared with max-pressure controls by Varaiya (11) and Le et al. (1) using a one-node network (7). They showed that both max-pressure controls failed to maximize the throughput but P_0 policy could, but the result of this study might be restricted by the structure of the test network. As far as we know, there is no study testing the performance of the max-pressure control in a large network considering road users' route choice behaviors and showing the result of traffic assignment with the max-pressure control. In this study, the existence of user equilibrium in the network with the max-pressure control is proven and the traffic assignment result under the user equilibrium was calculated in simulation.

2 Network model

Consider a network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ with a node set $\mathcal{N}$ and a link set $\mathcal{A}$. Let $\mathcal{Z} \subseteq \mathcal{N}$ be the set of zones. Links are divided into ordinary links $\mathcal{A}_{\mathrm{o}}$ and entry links $\mathcal{A}_{\mathrm{z}}$. Entry links are source links use which vehicles enter the network. Ordinary links are links that are not connected with any source node or sink node. We assume that the free flow travel time of each link is 1 time step. Let $d_{rs}(t)$ be the demand from zone r to zone s at time step t. $\mathbf{d}(t)$ is a set of $d_{rs}(t)$. Assume $\mathbf{d}(t)$ are i.i.d. with respect to time with mean $\bar{\mathbf{d}}$.

Let $x_{ij}(t)$ be the number of vehicles on link i waiting to turn to link j at time step t. The state at time t is the queue lengths $\mathbf{x}(t)$. Let $\mathcal{X} = \mathbb{Z}_+^{|\mathcal{A}|}$ be the set of all states. Let $y_{ij}(t)$ be the number of vehicles actually turning from i to j at time step t. Let $p_{jk}(t)$ be the probability that a vehicle entering j will move to k at time step t. Assume that $\mathbf{p}(t)$ are i.i.d. with respect to time with mean $\bar{\mathbf{p}}$. The mean is determined by route flows. The queue at time step t evolves via:

$$x_{jk}(t+1) = x_{jk}(t) - y_{jk}(t) + \sum_i y_{ij}(t) p_{jk}(t) \tag{1}$$

with $y_{ij}(t)$ limited by the capacity and signal activation:

$$y_{ij}(t) = \min\{x_{ij}(t), s_{ij}(t) C_{ij}\} \tag{2}$$

where $s_{ij}(t)$ is the signal activation. Like Varaiya (11) (and static traffic assignment), we assume that turning movements have a hard capacity constraint of C_{ij} but unbounded queues.

From (11), we can define the stable region of the max-pressure control as follows: Let f_{ij} be the average flow rate from i to j. Let q_i be the average flow entering link $i \in \mathcal{A}_{\mathrm{z}}$. Given $\bar{\mathbf{p}}$, f_{ij}

must satisfy

$$f_{jk} = \sum_i f_{ij}\bar{p}_{ij} \qquad \forall (i,j) \in \mathcal{A}_{\mathrm{o}}^2 \tag{3}$$

$$f_{jk} = q_j \bar{p}_{jk} \qquad \forall (j,k) \in \mathcal{A}_{\mathrm{z}} \times \mathcal{A}_{\mathrm{o}} \tag{4}$$

For any entering flow rates $\mathbf{q}$ and turning proportions $\bar{\mathbf{p}}$, $\mathbf{f}$ can be uniquely determined. The stable region of the network, $\mathcal{Q}$, is the set of entering flows $\mathbf{q}$ such that there exists an $\bar{\mathbf{s}}$ satisfying $f_{ij} \leq \bar{s}_{ij} C_{ij}$ where C_{ij} is the capacity for the turning movement from i to j. Let $\mathcal{Q}^0$ be the interior of $\mathcal{Q}$, i.e. where $f_{ij} < \bar{s}_{ij} C_{ij}$. Varaiya (11) proved that if $\mathbf{q} \in \mathcal{Q}^0$, using the max-pressure control would lead to bounded queue lengths, i.e.

$$\lim_{T\to\infty} \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{A}^2} x_{ij}(t) \leq \kappa < \infty \tag{5}$$

In the max-pressure control, the intersection controller activates the phase with the largest pressure $(\sum_{ij} w_{ij} s_{ij} C_{ij})$. The weight of a turning movement in the max-pressure control is defined as:

$$w_{ij}(t) = x_{ij} - \sum_k x_{jk}(t) p_{jk}(t) \tag{6}$$

3 Equilibrium

When traffic assignment is incorporated in the model, road users change paths based on experienced travel times and the turning proportion $\bar{\mathbf{p}}$ and entering flow $\mathbf{q}$ could change. Let Π be the set of paths, with $\Pi_{rs} \subseteq \Pi$ be the set of paths from r to s. Let $\delta_i^\pi \in \{0,1\}$ indicate whether path π uses link i. Let $h^\pi \geq 0$ be the average flow on path π. Then f_{ij} is related to path flow h^π with:

$$f_{ij} = \sum_{\pi\in\Pi} h^\pi \delta_i^\pi \delta_j^\pi \tag{7}$$

Path flows also determine turning proportions:

$$\bar{p}_{ij} = \frac{\sum_{\pi\in\Pi} h^\pi \delta_i^\pi \delta_j^\pi}{\sum_{\pi\in\Pi} h^\pi \delta_i^\pi} \tag{8}$$

which is the conditional probability that a vehicle will use link j given that it uses link i. The entering flow is given by

$$q_i = \sum_{\pi\in\Pi} h^\pi \delta_i^\pi \tag{9}$$

Combined with the standard constraint that relates path flows to demands between origin-destination pairs

$$\sum_{\pi \in \Pi_{rs}} h^{\pi} = \bar{d}_{rs} \tag{10}$$

the path flows are related to average link flows in the max-pressure stability region. Let $\mathcal{H}$ be the set of path flows satisfying constraint (10).

Let $\bar{x}_{ij} = \lim\limits_{T\to\infty} \sum\limits_{t=1}^{T} x_{ij}(t)$ be the average queue length for (i,j). Let τ_{ij} be the average travel time for turning movement (i,j). From Little's Law,

$$\bar{x}_{ij} = \tau_{ij} f_{ij} \tag{11}$$

which gives a formula to calculate τ_{ij} given $\bar{x}_{ij}$.

We define user equilibrium as usual: vehicles choose minimum-travel time routes. Equivalently, that can be written as

$$h^{\pi}(\tau^{\pi} - \mu_{rs}) = 0 \tag{12}$$

where $\tau^{\pi} = \sum\limits_{ij} \tau_{ij} \delta_i^{\pi} \delta_j^{\pi}$ is the travel time for path π and $\mu_{rs} = \min\limits_{\pi \in \Pi_{rs}} \{\tau^{\pi}\}$.

3.1 Existence of equilibrium

Let $\bar{\mathbf{X}}(\mathbf{h})$ be the average queue lengths under path assignment $\mathbf{h}$. Once $\bar{\mathbf{X}}(\mathbf{h})$ is known, then travel times for every link are determined. Given fixed travel times $\mathcal{T}\left(\bar{\mathbf{X}}(\mathbf{h})\right)$, let $H(\tau)$ be the set of path assignments satisfying user equilibrium for the travel times. Since $\tau = \mathcal{T}\left(\bar{\mathbf{X}}(\mathbf{h})\right)$ is a function of the path assignment $\mathbf{h}$, we can write $H(\cdot)$ as the composition of several functions depending on $\mathbf{h}$, i.e. $H : \mathcal{H} \to 2^{\mathcal{H}}$. Notice that H is a set-valued function; given fixed travel times, there may be multiple path assignments that are equilibrium. From that definition, H is a multifunction. A path flow assignment $\mathbf{h}$ is an equilibrium if and only if $\mathbf{h} \in H(\mathbf{h})$, i.e. no vehicle has an incentive to change paths given the travel times resulting from $\mathbf{h}$. Based on the Kakutani's fixed point theorem, we prove the existence of the fix point of $H(\mathbf{h})$.

Since $H(\mathbf{h})$ has a fixed point, then there exists at least one user equilibrium but it is not clear that equilibrium is unique.

4 Simulation

To validate the existence of a user equilibrium in a network with the max-pressure control, we use simulations to get the traffic assignment result. Figure 1 shows the algorithm to get the result of the traffic assignment under the user equilibrium with varying turning proportions under the max-pressure control. At each time step, each intersection calculates the weight of turning movements based on the turning proportion and the current queue length. The phase with the largest weight is activated. As the intersection control affects vehicle delays at intersections and further affects the total travel times, some vehicles switch their paths to the shortest path in the following iteration. The simulation updates the turning proportion of each turning movement every iteration based on the path assignment result, and the simulation stops when the criterion for convergence is satisfied. Figures 2 and 3 shows the variation of the ratio of actual travel time and the free-flow travel time under moderate and high demands. The number of congested links reduces when the number of iterations increases.

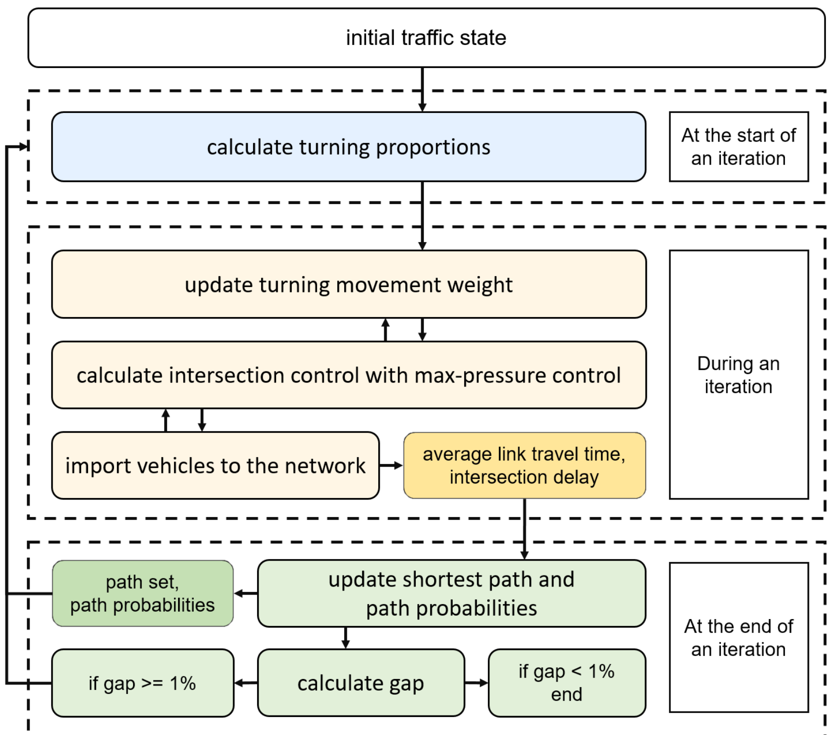

Figure 1: Simulation process with the max-pressure control

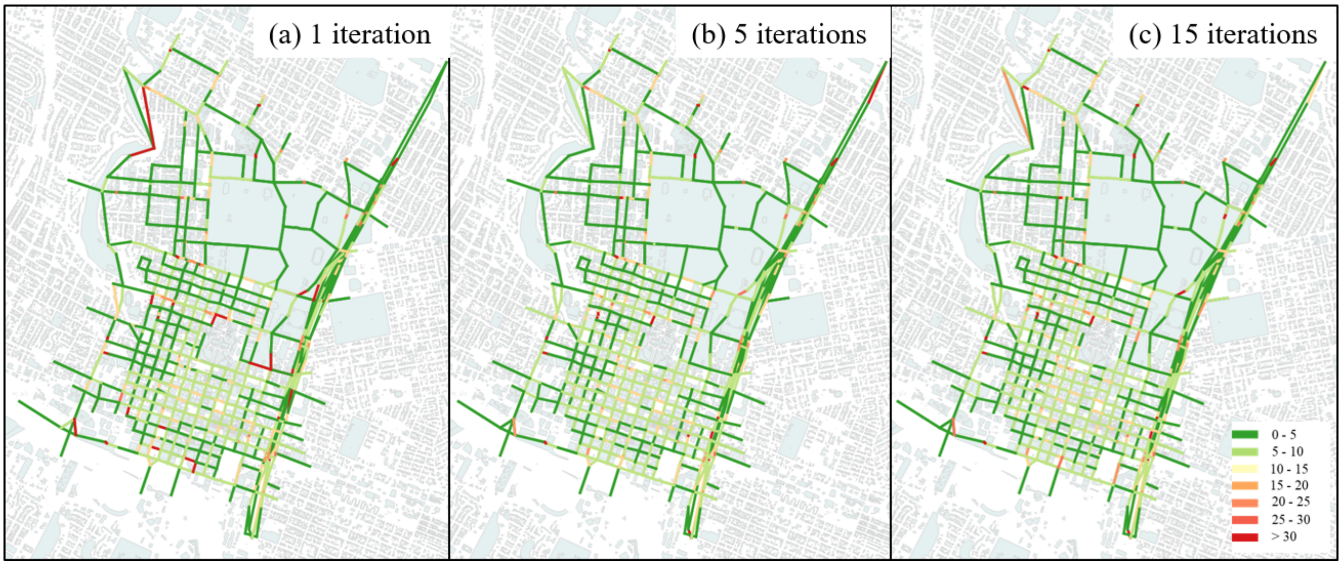

Figure 2: Variation in the ratio of actual travel time and free-flow travel time with moderate demand

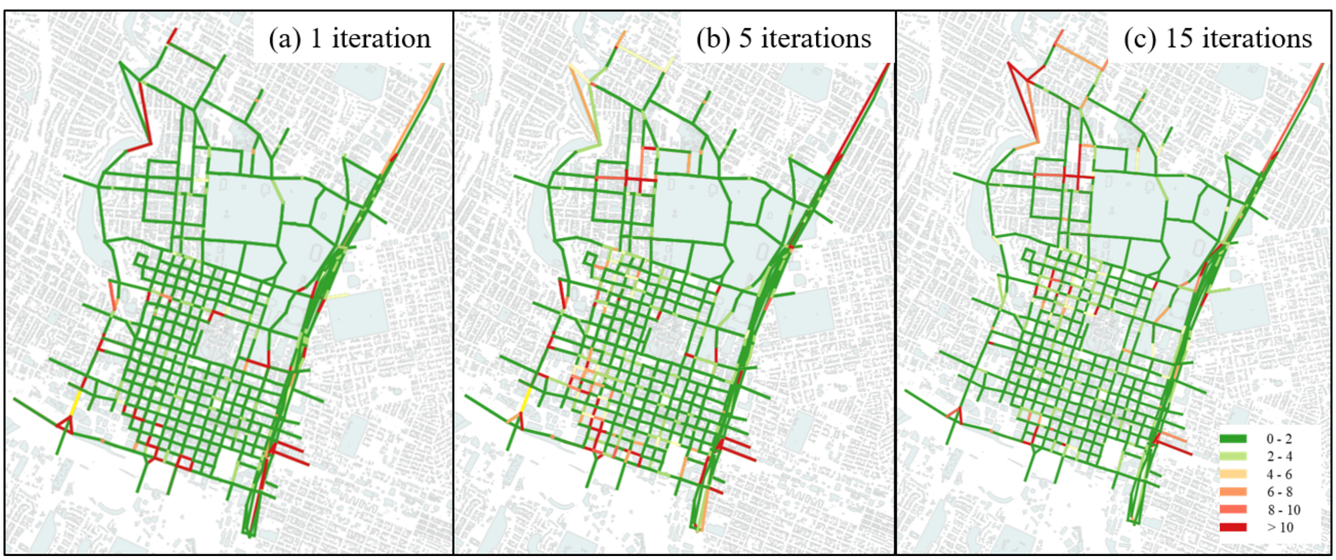

Figure 3: Variation in the ratio of actual travel time and free-flow travel time with high demand

5 Conclusion

Existing studies about the max-pressure control assume fixed turning proportions, but this study tests the effect of the max-pressure control considering the impact of route choice behaviors. In this study, the existence of the user equilibrium is proved using the Kakutani's fixed point theorem. The simulation results show that when the iteration number increases, there are fewer congested roads.

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