A Model for System Optimum Dynamic Traffic Assignment with Minimum-Envy Allocations

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Keywords: Dynamic Traffic Assignment, Dynamic System Optimum, Envy, Fairness, and Transportation as a Service

INTRODUCTION

With real-time information having become readily available, dynamic traffic assignment models can assume an optimal traveler behavior where travelers select the fastest travel option based on current traffic conditions (greedy assumption), known as the Dynamic User Optimum (DUO). This assumption is valid when we only consider the availability of real-time traffic information. However, we can postulate that this is not a dynamic system-optimal solution (DSO) since greedy behavior does not improve the efficiency of a system.

A service provider in the future who designs “transportation as a service” will devise a platform that maximizes the efficiency of a system while minimizing agents’ complains from unfairness. When a system allows travelers to exchange their travel information with others, travelers might feel envious of others who have a faster route. Also, the feeling of unfairness might differ by drivers concerning his/her preference.

To model a fair and efficient allocation of transportation supply for heterogeneous travelers, we employ Envy models which were initially employed for a fair division problem in the context of the fair cake-cutting problem in the 1940s (Barbanel 2005). Envy-Freeness implies that each agent believes that their allocation is greater than or at least the same as the share of others. Each player, in turn, is satisfied with an allocated piece of cake according to their preference. Thus, an EF region exists if there is a certain level of the heterogeneity of preference(Gamow and Stern, 1958, Varian, 1974).

Our goals regarding dynamic cases are maximizing system efficiency while minimizing agents’ envy. This chapter will address the following questions: 1) how to find the best alternatives under dynamic traffic demands and conditions, 2) how unfairness is evoked by efficient transportation, 3) how to obtain the best pricing policy for minimizing envy.

METHODOLOGY

Future mobility systems might include shared, connected and autonomous components to various extents as smart systems. More specifically, we focus on the optimization of the allocation problem to achieve both system-wide efficiency and minimum envy among individuals. We model “transportation as a service” which accounts for individual level of allocation.

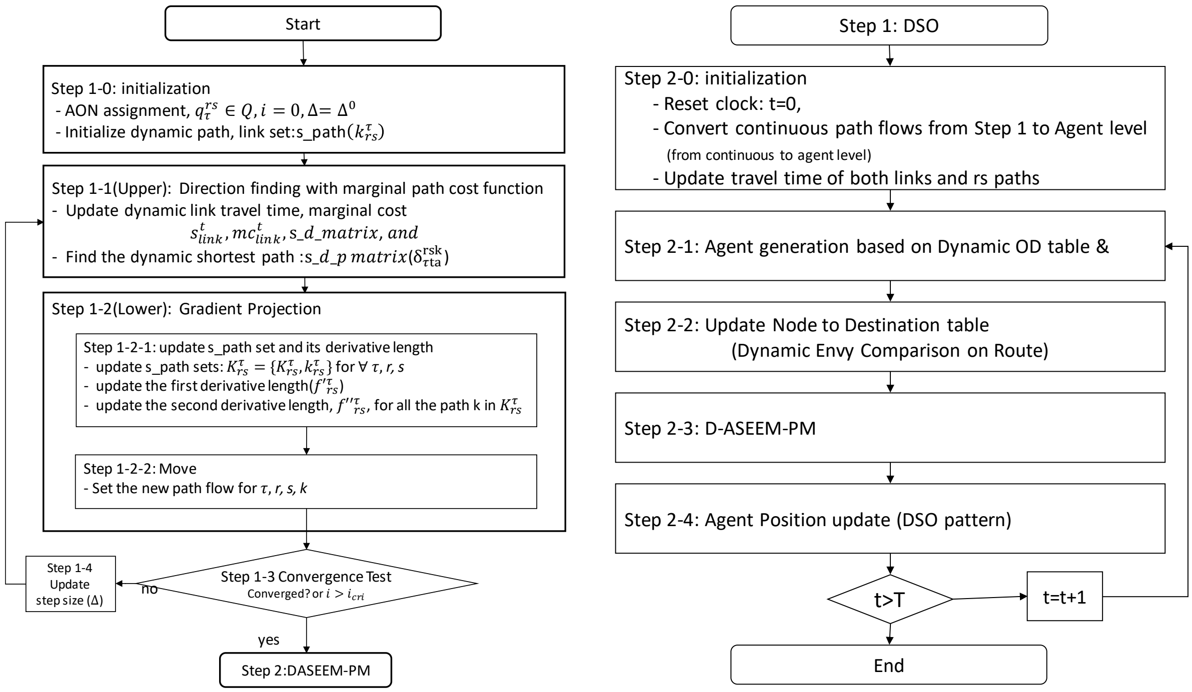
Traffic varies by time of day. Furthermore, as agents travel on their route, their envy may differ according to time dynamics (such as traffic conditions). Any pricing scheme, therefore, needs to account for this variance. In other words, the actual application of the proposed model needs to implement time-space dynamics.

The proposed model dynamically allocated system efficiency with an envy minimization-price match (DASEEM-PM). The objective of the DASEEM-PM problem is as follows.

$$\min Z = \alpha \sum_{t \in T} \sum_{a \in A} x_{a,t} t_{a,t}(x_{a,t}) + \beta \sum_{i \in I} \max_{i \neq j} \{ {e^{rs}}_{ij} \} \tag{1}$$

The first term of the objective function is associated with the system optimum that is a minimized travel time. The associated constraints close to constraints of the dynamic system optimum. The fundamental idea that undergirds this research is that the DSO conditions have more diverse routes compositions than the corresponding UE condition. The second term of the objective function is to minimize envy induced from the travel time gaps among diverse routes in the same OD pair. Based on the computed patterns, we can solve the efficient route allocation problem with the consideration of pricing trade-off among agents in the same origin-destination pair.

The requirements for the second objective are path-based flow patterns. Thus, we develop a path-based system optimum solution by applying a Gradient Descent Projection method and a time-dependent network structure by referring to Jayakrishnan, Tsai, and Chen (1995) and Yang (2011),shown in Figure 1. It is noteworthy that the proposed method considers the discrepancy between the marginal cost and link travel time. In other words, an actual position of a vehicle at time step t should be updated based on not marginal cost but link travel time even if a system optimum assignment.



(Step 1) GP-based Bi-Level programming solution for DSO

(step 2) Solution for the second objective

Constraints for Peer-to-Peer envy comparison are formulated as from Eq (*2)* to Eq (*4)*. Eq (2) addresses envy comparison. It indicates that if agent i pays more than agent j ($p_i \geq p_j$) for the same path travel time, agent i feel envious (e_{ij}) at the amount of the price difference. Similarly, if the path travel time of agent j is shorter than that of agent i without price difference ($p_i = p_j$), agent i feels envious (e_{ij}) to agent j. It is noteworthy that the feeling of envy is only measured by agent i's valuation θ_i. In Eq (3), $\sum_{r,s,k,\tau} -t_a \delta_{it\tau a}^{rsk}$ is the path travel time of path k for agent i. Thus, envy in the latter case, can be interpreted as a degree of travel time difference as perceived only by agent i. Eq (4) is a budget constraint for an OD and time pair

$$\sum_{r,s,k,\tau} -t_a \delta_{it\tau a}^{rsk} \theta_i + e_{ij} - p_{i\tau}^{rs} \geq \sum_{r,s,k',\tau} -t_a \delta_{jt\tau a}^{rsk'} \theta_i - p_{j\tau}^{rs} \qquad \forall\, i, j \in I,\ i \neq j \qquad (2)$$

$$0 \leq e_{ij} \leq e_{max} \qquad \forall\, i, j,\ i \neq j \qquad (3)$$

$$\sum_{i \in I} p_i^{rs\tau} = B^{rs\tau} \qquad \forall\, r, s, \tau \qquad (4)$$

RESULTS

We examine the proposed method on a hypothetical network (a dynamic version of Braess Paradox Network), as shown in Table 1. Initially, a single origin-destination case shows an example of how routes vary under the DSO condition and the proposed pricing scheme plays a role in minimizing envy induced by allocations for DSO.

Table 2 provides a numerical summary of the performance of each scenario. Total maximum envy is minimized to zero in DASEEM-PM, which supposed to be 41,348 units in DASEEM. This implies that without a pricing scheme, assigning agents to SO routes induces significant envy. Among agents who have the same origin and destination and the same departure time, some agents might feel satisfaction when they arrive at their destination earlier than others'. Agents who matched to the fastest path group in DASEEM might feel more benefit than agents in DUE since the travel time gaps of DSO between routes tend to be substantial. Whereas agents matched to longer paths feel envy, agents belonging to the shortest path might feel benefit. Note that the Allocation Efficiency function in DASEEM matches the paths with respect to the agents' valuations. Consequently, DASEEM minimizes total envy while maximizing total monetary benefit. However, DASEEM could still not be considered as a feasible application because of unfairness (envy). However, the pricing scheme in DASEEM-PM minimizes envy to 0 and maximizes monetary benefit that agents perceive from P2P travel option comparisons.

Table 1 Link characteristics of Braess's Paradox network

Link ID	Head node	Tail node	Jam density (veh/mile)	# of lanes	Length(ft)	Speed(ft/sec)
a	1	2	160.0	6	6000.0	51.33
b	1	3	160.0	6	3000.0	51.33
c	2	3	160.0	6	900.0	51.33
d	3	2	160.0	6	900.0	51.33
e	2	4	160.0	6	6000.0	51.33
f	3	4	160.0	6	6000.0	51.33

Table 2 Performance Comparisons between Models

Index	DUE	DASEEM(wo PM)	DASEEM-PM
Total Travel time (seconds)	1.62E+06	1.58E+06 (-2.09%)	1.58E+06 (-2.09%)
Total travel miles (ft)	6.75E+07	6.94E+07 (2.74%)	6.94E+07 (2.74%)
Average Speed(SMS,mph)	28.46	29.87 (4.95%)	29.87 (4.95%)
Total max Envy (a)	5.68E+03	4.13E+04 (627%)	0.00 (-100%)
Total max Monetary Benefit (b)	1.55E+04	1.44E+05 (825%)	9.29E+04 (497%)
Net satisfaction (b)-(a)	9.87E+03	1.02E+05 (938.45%)	9.29E+04 (841.13%)
Total Transaction	-	-	6.44E+04

CONCLUSIONS

Future technologies will require smart management of urban infrastructure systems. Our primary research interest is to develop a mobility platform for Transportation as a Service for achieving both fairness and efficiency, goals that often compete with each other. We focus on the design of a transportation planning platform which maximizes system efficiency while minimizing user 'envy'. For a real-world implementation application, we model the spatiotemporal dynamics of traffic conditions on the network using Dynamic System Optimum Assignment, considering the time constraints of route guidance. Traditional planning methods limit the demand characteristics as an aggregated behavior, meaning that all travelers have identical behavior. Even efforts to consider the heterogeneity of travelers only focus on broad categorical travelers who are assumed to have the same behavior within their group. To solve this problem, we have developed extensions to the envy-free allocation theory, an idea from Economics. Our solution arrives at a pricing scheme, achieving both Pareto efficiency and fair allocations by considering an individual-level preference and efficient allocation of supply.

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