

1 Optimal Assignment and Relocation of Shared Autonomous
2 Vehicles Considering Mode Choices

3 Yang Liu *

4 Department of Civil and Environmental Engineering
5 & Department of Industrial Systems Engineering and Management
6 National University of Singapore, Singapore
7 Email: iseliuy@nus.edu.sg
8 Phone: 65-6516-2334

9 Hao Guo

10 Department of Industrial Systems Engineering and Management
11 National University of Singapore, Singapore
12 Email: E0154200@u.nus.edu
13 Phone: 65-9613-7839

14 Yao Chen

15 Department of Industrial Systems Engineering and Management
16 National University of Singapore, Singapore
17 Email: isechy@nus.edu.sg
18 Phone: 65-8678-8546

19 *Keywords: Autonomous vehicle sharing systems; Discrete choice model; Elastic demand; Piece-wise*
20 *linear approximation*

*Corresponding author

1 Introduction

It is predicted that Autonomous Vehicles (AVs) will enter the global vehicle market in the next few decades. The expected benefits associated with AVs, including increased safety and reduced traffic and parking congestion, may not be significant unless the vehicles become affordable and common (Litman, 2017). In the meanwhile, sharing mobility services will continue to grow, which greatly improves the vehicle and parking utilization rates. Therefore, it is desirable to develop efficient Shared Autonomous Vehicle (SAV) systems to make AVs more affordable and accessible.

In the SAV systems, operators need to make efficient operational decisions to meet the needs of travel demand in both spatial and temporal dimensions. Efficient vehicle operations aim to generate high profits. Meanwhile, it also requires a high level of service to attract sufficient demand requests. Compared to Conventional Private Vehicles (CPVs), the mobility services provided by efficient SAV systems may be more convenient and flexible thanks to self-driving technologies. This paper focuses on the optimal assignment and relocation problem of SAV systems, while the competitions between SAVs and CPVs are explicitly modeled by a discrete choice model, in which the attributes such as travel time, travel cost, and level of service are considered to determine the market share of SAVs.

Many studies have been conducted in the field to address the dispatching, fleet sizing, and pricing problems in vehicle sharing operations. Most of them focus on the vehicle relocation problem. The relocation of shared vehicles can be completed actively by relocation staff operations (e.g., Kek et al., 2006) or passively by adopting pricing strategies to influence demand (e.g., Barth et al., 2004 and Xu et al., 2018). The demand for transportation modes can be influenced by many factors. The elastic demand has been considered in the car-sharing literature, which mainly focus on the effectiveness of pricing strategies to rebalance the system (e.g., Jorge et al., 2015). These studies consider the influence of pricing on car-sharing demand in car-sharing relocation problems. There is a limited number of empirical studies that consider the impact to demand from other factors such as travel time. Catalano et al. (2008) applied the multinomial logit (MNL) model that revealed the competition of car-sharing service with private vehicles, carpooling, and public transit. Zhou and Kockelman (2011) also adopted the MNL to predict the likelihood of choosing car-sharing as a travel mode among the existing ones. However, there are few studies using the passive relocation examine the effect of factors other than pricing. For example, Huang et al. (2018) study an optimal station location problem of car-sharing with mode choice and non-linear demand affected by travel time. In our study, we take into account the impacts from travel costs, travel times, and level of service to SAV demand. The level of service is represented by the availability of vehicles and availability of parking spaces.

2 Methodology

We examine the vehicle assignment and relocation problem for one-way SAV systems. To determine the optimal vehicle assignment and relocation plan, we propose a time-space network flow model, which is formulated as a non-linear mixed-integer program. A binary logit discrete choice model is incorporated into the optimization program to capture travelers' mode choices between

1 SAVs and CPVs. The proposed non-linear mixed-integer program is computationally challeng-
2 ing and expensive. To make this problem tractable, we first reformulate the original model to
3 make the logarithmic functions the only non-linear constraints. A piece-wise linear approxi-
4 mation method is developed to linearize the non-linear constraints. Furthermore, the number
5 of break-points has significant effects on the solution quality and efficiency. More break-points
6 leads to a tighter linear approximation so that the piece-wise linear function can have any degree
7 of accuracy (see, e.g., Wang and Lo (2010); Liu and Wang (2015); Wang et al. (2015)). However,
8 introducing a large number of break-points will significantly increase the computation burden.
9 Therefore, the proper break-points should be carefully selected to achieve a good approximation
10 within an acceptable computation time. We propose a dynamic programming to determine the
11 optimal break-point selections. By applying this approach, we show that the proposed solution
12 approach can improve the approximation accuracy without increasing too much computation
13 requirement.

14 **3 Main Results**

15 We apply our approach to real-world cases based on the city of Singapore collected from a car-
16 sharing company BlueSG. In this data set, a total of 10 stations are selected. We set the SAV
17 fleet size from 100 to 400 and the demand from 1000 to 5000 in order to create 20 scenarios. For
18 each scenario, five demand instances are generated from a given probability distribution using
19 simulation. Our numerical findings are summarized as follows:

- 20 • The computational results show our approach can consistently produce satisfactory solu-
21 tions in all instances. We use a case with four times evenly distributed break-points as the
22 benchmark. By using the dynamic programming, the largest objective value gap between
23 our approach and the benchmark is less than 1%. Moreover, the computation time of our
24 approach reduced by 90 % compared with the time of the benchmark on average.
- 25 • Under our optimal assignment and relocation policies, there is more relocation activities
26 when the demand pressure is moderate. Consequently, it leads to a high fulfillment rate and
27 market share of SAVs. Few relocations have been conducted when the demand pressure is
28 extremely low or high.
- 29 • We replace the SAVs in the original problem with the Shared Conventional Vehicles (SCVs)
30 to study the impact of vehicle types. SAV services of high price may have a high fulfillment
31 rate when the demand pressure is high. In both median and low price cases, the SCV
32 systems show less vehicle utilization. When the demand pressure is extremely low or high,
33 the gap between cases with SAVs and SCVs is small.
- 34 • After considering the daily cost of SAVs in the objective function, the maximum profit is
35 achieved when the total demand is 10 to 15 times the SAV fleet size. A moderate demand
36 pressure is the most suitable for SAV systems. Too high or too low demand will lead to a
37 decrease in profits.
- 38 • We generate different demand sets to study the influence of city size and travel distance
39 distribution. The SAV services have less fulfillment rate and less market share in a larger

1 city. Nevertheless, demand sets with more short travel requests lead to a higher fulfillment
2 and lower utilization of SAV systems.

- 3 • We test the impact of demand symmetry to SAV systems. There are more relocation ac-
4 tivities when the demand is asymmetric compared to symmetric ones in all scenarios. But
5 the traffic efficiency is still not as good as that under symmetric demand. Given the same
6 demand quantity, the profit under an asymmetric demand set is lower than under a sym-
7 metric case.

8 **4 Conclusion**

9 In this paper, we address the optimal SAVs operation problem with competition from CPVs. We
10 propose a solution approach for solving the original model. A piece-wise linear approximation
11 method is developed to linearize the non-linear constraints. Further, a dynamic programming
12 is developed to select optimal break-points. We show that the proposed solution approach can
13 consistently and efficiently obtain optimal solutions through quantitative computational experi-
14 ments. With optimal assignment and relocation decisions, the SAVs may outperform the CPVs.
15 Numerical results reveal that when the demand pressure is moderate, SAVs can achieve a higher
16 fulfillment rate and market share due to more relocation activities. Furthermore, we study the
17 optimal fleet size and find that the maximum profit is achieved when the demand pressure is
18 moderate. We also evaluate the influence of demand patterns as well as city sizes. The results
19 show that SAV services may be preferable in a small size city with more short-distance and
20 asymmetric travel demand. We may extend this work in future studies by considering stochastic
21 nature in transportation systems or a dynamic case.

22 **Acknowledgements**

23 The work was supported by the Singapore Ministry of Education Academic Research Fund
24 Tier 1 (R-266-000-135-114) and Singapore Ministry of Education Academic Research Fund Tier 2
25 (MOE2017-T2-2-128).

26 **References**

- 27 Barth, M., Todd, M., and Xue, L. (2004). User-based vehicle relocation techniques for multiple-
28 station shared-use vehicle systems.
- 29 Catalano, M., Lo Casto, B., and Migliore, M. (2008). Car sharing demand estimation and urban
30 transport demand modelling using stated preference techniques.
- 31 Huang, K., de Almeida Correia, G. H., and An, K. (2018). Solving the station-based one-way
32 carsharing network planning problem with relocations and non-linear demand. *Transportation*
33 *Research Part C: Emerging Technologies*, 90:1–17.

- 1 Jorge, D., Molnar, G., and de Almeida Correia, G. H. (2015). Trip pricing of one-way station-based
2 carsharing networks with zone and time of day price variations. *Transportation Research Part B:
3 Methodological*, 81:461–482.
- 4 Kek, A. G., Cheu, R. L., and Chor, M. L. (2006). Relocation simulation model for multiple-station
5 shared-use vehicle systems. *Transportation research record*, 1986(1):81–88.
- 6 Litman, T. (2017). *Autonomous vehicle implementation predictions*. Victoria Transport Policy Institute
7 Victoria, Canada.
- 8 Liu, H. and Wang, D. Z. (2015). Global optimization method for network design problem with
9 stochastic user equilibrium. *Transportation Research Part B: Methodological*, 72:20–39.
- 10 Wang, D. Z., Liu, H., and Szeto, W. (2015). A novel discrete network design problem formulation
11 and its global optimization solution algorithm. *Transportation Research Part E: Logistics and
12 Transportation Review*, 79:213–230.
- 13 Wang, D. Z. and Lo, H. K. (2010). Global optimum of the linearized network design problem
14 with equilibrium flows. *Transportation Research Part B: Methodological*, 44(4):482–492.
- 15 Xu, M., Meng, Q., and Liu, Z. (2018). Electric vehicle fleet size and trip pricing for one-way
16 carsharing services considering vehicle relocation and personnel assignment. *Transportation
17 Research Part B: Methodological*, 111:60–82.
- 18 Zhou, B. and Kockelman, K. M. (2011). Opportunities for and impacts of carsharing: A survey
19 of the austin, texas market. *International Journal of Sustainable Transportation*, 5(3):135–152.