Estimation of Schedule Preference and Crowding Perception in Urban Rail Corridor Commuting: An Inverse Optimization Method

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This paper introduces an inverse optimization method to uncover commuters’ schedule preference and crowding perception based on aggregated observations from smart card data for an urban rail corridor system. The assessment of time-of-use preferences typically involves the use of econometric models of discrete choice based on detailed travel survey data. However, discrete choice models often struggle with potential endogeneity issues in behavioral observations when estimating individual samples from massive transit data with limited exogenous identifying information. This motivates us to employ an equilibrium modeling approach to capture the dynamism hidden in commuters’ departure time decision-making from aggregations. Assuming user optimality in observed choices, an inverse optimization method is proposed to find a set of preference parameters in the stochastic user equilibrium-based morning commuting model with heterogeneous commuters so that the resulting equilibrium pattern best approximates the observed departure rate distribution over time. The proposed inverse optimization problem can

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be formulated by a bi-level programming model and a sensitivity analysis-based solution framework is further designed for model estimation. Lastly, the smart card data and train timetable data from the rail corridor along the Beijing Subway Batong Line are synthesized for a case study to estimate commuters’ departure time choice preferences during morning peak periods, as well as to validate the robustness and practicality of the proposed method.

1 Introduction

The urban rail transit systems play an important role in urban transportation, mitigating road traffic congestion by encouraging a shift from private car usage to public transportation, thereby enhancing the efficiency of urban transportation networks. Usually, the rail transit lines are planned, designed and operated along corridors with dense passenger flows, forming rail transit corridors as the backbone of the urban transportation network. Consequently, the issue of in-train or in-station overcrowding caused by large passenger flows during peak hours may lead to severe safety concerns. For urban rail transit commuters, the additional travel costs due to prolonged overcrowding will have a significant impact on their travel behaviors/choices, especially their choices of departure time. In this context, proper understanding and prediction of morning commuters’ departure time choice behavior in urban rail transit corridors are crucial for effective commuting demand management and efficient mass transit operations.

The analysis of commuters’ time-of-use decisions underpins the modeling of urban rail transit commuters’ departure time choice behavior. Commuters are often assumed to determine their respective optimal departure time that best balance time-dependent marginal rates of utility derived from different locations. Vickrey (1973) formulated the departure time choice problem by minimizing the sum of two integrals, where the integrands represent the time-dependent marginal utilities of time at home and at the office/work relative to the time spent in travel. The well-known ‘$\alpha - \beta - \gamma$’ scheduling formulation, proposed by Vickrey (1969) and further developed by Small (1982), is a special case of the model introduced by Vickrey (1973), where marginal time utility is $\alpha$ at home, and $\alpha - \beta$ before desired arrival time and $\alpha + \gamma$ afterwards at the workplace. This constant time utility specification has become one of the most important references for studying the valuation of travel time in road commuting, given its simplicity and tractability. In addition to relatively measurable metrics such as travel time and schedule delays (similar to road traffic modeling), for mass transit systems, the in-vehicle crowding, mainly attributed to the privacy loss, body touch (uncomfortable physical proximity), air pollution and venture of the pickpocket (Huang, 2000; Tian et al., 2007a), is another important factor impacts commuters’ choice behavior.

The value, cost, or disutility of in-vehicle crowding has been empirically evaluated in different ways based on survey data. Li and Hensher (2013) discovered a significant disparity between objective and subjective measures of crowding by conducting a thorough review of existing evidence. They suggested that the inclusion of subjective metrics can yield a more accurate representation of crowding, which should be examined through a preference study. Haywood and Koning (2015) reported a near-linear relationship between crowding costs and density from individuals’ stated preference data for the Paris subway. Tirachini et al. (2016) estimated differences in the valuation of travel time sitting and
standing by modeling the observed choices of metro commuters who prefer to first travel backwards to secure a seat and then travel forward in Singapore. Hörcher et al. (2017) presented a method to estimate the cost of crowding in terms of the equivalent travel time loss in a revealed preference route choice framework and suggested that the stated choice methods may overestimate the cost of crowding. Many studies on crowding valuation also incorporated other aspects such as mode choice, crowding externalities, physical and human factors, international comparison as well as the effects of COVID-19 in recent years (Tirachini et al., 2013, 2017; Márquez et al., 2019; Wang and Zacharias, 2020; Aghabayk et al., 2021). However, all these works were carried out in the context of static models and are unable to capture the morning transit commuters’ time-of-day choices that are critical input for optimizing travel demand management strategies. Based on a stated preference survey conducted in Beijing, Yan et al. (2016) developed and calibrated an urban rail departure time choice model that incorporated various factors such as travel time, schedule delay, in-vehicle crowding and subway fare. Similarly, based on survey data for the Beijing subway system, Li et al. (2018) investigated various attributes that impact urban rail passengers’ departure time choice and developed a mixed logit model of departure time choice that accounts for price endogeneity. Similar studies have been carried out for the Nanjing subway, where Cheng et al. (2020) explored the heterogeneity of passengers’ departure time choice behavior during peak hours. Lizana et al. (2021) further formulated and estimated a joint travel mode-departure time model for commuting trips using the combined revealed preference and stated choice data.

Empirical results obtained through the sophisticatedly formulated econometric models of discrete choice were often directly used to evaluate users’ generalized travel costs. These results were subsequently employed in cost-benefit analysis for evaluating transport projects or policies, which followed a “firstly estimate, then evaluate/equilibrate/optimize” framework. In general, this estimation process tends to prioritize the minimization of estimation error without adequately considering the applicability of the estimated values for subsequent analysis or optimization problems. The increasing availability of extensive trip data from, e.g., GPS or Smart Card Automatic Fare Collection System, allow the users’ repeated and regular travel behavior to be observed, which is theoretically superior to the stated preference data. However, the observed travel choice data often involves inherent limitations where there is limited external knowledge about individuals and their decision-making environment. This can yield highly correlated (and thus biased) outcomes (Small, 2012). When certain observed preference metrics correlate with the error term in an econometric model, endogeneity arises. This can be attributed to issues such as omitted variables, measurement or specification errors, simultaneous determination or self-selection (Guevara, 2015). The presence of endogeneity in a model can potentially induce estimation bias, leading to inaccuracies in the estimated parameter values or even misleading results.

In this paper, we make an attempt to estimate commuters’ schedule preference and crowding perception via econometric models of discrete choice using smart card-based mobility data from a many-to-one urban rail corridor system (mainly presented in Section 5.1). Given that certain measurements of in-vehicle crowding degree from system-level aggregations are the outcomes of individual choices, employing these endogenous statistics to estimate discrete choice models can introduce not only significant bias but also incorrect signs. The sign bias in the estimation of the crowding perception parameter gives rise to an empirical paradox, suggesting that commuters are more willing to depart/travel when the train is more crowded during morning peak period. The same endogeneity issue is also recognized by Peer et al. (2016) when evaluating the effect of crowding in trains based on revealed preference data, while not resolved. To address such endogeneity issues, the classical control-function method and latent-variable approach, as well as their combinations and
variants, are classical strategies widely utilized (Guevara and Ben-Akiva, 2010; Guevara, 2015). However, constructing compelling instrumental variables or latent variables from limited behavioral information in observations, such as passenger’ smart card-based tap-ins and tap-outs spatiotemporal records, can be extremely challenging and sometimes even infeasible. Solely relying on smart card-based observations, some studies aim to mitigate the endogeneity of in-vehicle crowding by investigating passengers’ riding behaviors in the context of quasi-experiments. For instance, some commuters are willing to travel backward (Tirachini et al., 2016) or wait for subsequent trains to secure a seat and avoid in-vehicle crowding (Chen et al., 2023; Xu et al., 2023). However, these models are confined to discerning individual utility levels exclusively within specific commuting scenarios, thereby lacking applicability to general cases. To address endogeneity resulting from the existence of simultaneity, a common approach involves introducing a time lag between independent and dependent variables (Train, 2009; Zaefarian et al., 2017). Leveraging access to individuals’ day-to-day repetitive choices recorded in smart card data, we further attempt to use one-day lagged measurements of preference ingredients as references for commuters’ decisions, as presented in Section 5.1. Although examining individuals’ subtle behavioral avoidance of the previous day’s crowding can help mitigate sign bias in parameter estimates, the inherent sampling bias in individual-level observations (Guarda and Qian, 2023), along with the initial condition problem (Heckman and Singer, 1986), can still lead to inaccurate estimations.

While direct measurements of static observations often encounter challenges in mitigating potential correlations between individual choices and specific preference ingredients, equilibrium modeling methods are potential of capturing the endogeneity of decision-making and simulating the dynamic trade-off process among various cost measurements (see Appendix D and Figure 12 for more reference). The rail commuting equilibrium model is essentially a system of simultaneous equations that account for the dynamic interactions among commuters’ travel choices. Correspondingly, the iterative solution procedure for the equilibrium can be regarded as an evolutionary dynamic decision-making process that account for the lagged effects of travel costs and behavioral inertia, which ultimately converges to a specific equilibrium departure time choice pattern based on the given utility preferences. Therefore, we turn to calibrating preference parameters in a departure time choice equilibrium model by leveraging aggregated behavioral observations. Huang, Tian, and their coauthors were the first to develop a morning peak-period commuting model considering in-vehicle crowding and schedule delay early or late, and investigate the equilibrium properties on one-to-one and many-to-one mass transit systems, respectively (Huang et al., 2005, 2007; Tian et al., 2007a, 2007b, 2009, 2021; Wu and Huang, 2014). de Palma et al. (2015, 2017) further discussed the possible function forms for modeling crowding within public transit commuting and its implications for optimal transit service analysis. However, all these works were built on deterministic user equilibrium, assuming homogeneous commuters with identical desired arrival times and choice sets of departure times, and thus are unable to capture the randomness and heterogeneity in commuters’ real-world choices.

Estimation of parameters in an equilibrium model from observation data mainly involves solving an inverse optimization problem. This approach seeks to identify the model parameters that would render a given a candidate solution as (approximately) optimal (Ahuja and Orlin, 2001; Heuberger, 2004; Iyengar and Kang, 2005; Aswani et al., 2018; Chan et al., 2023). Specifically in this paper, we aim to calibrate the preference parameters of a rail corridor commute equilibrium model so that the equilibrium solution matches the actual observations as closely as possible. The calibration process is usually formulated as bi-level programming or mathematical programming model with equilibrium constraints. For example, Lam and Huang (1992) calibrated the weight parameters of the combined trip distribution and assignment model for multiple user classes using complete and partial observed traffic
information, respectively. Yang et al. (2001) proposed a bi-level programming model for simultaneous estimation of an origin-destination (OD) matrix and a travel-cost coefficient for congested networks in a logit-based stochastic user equilibrium from observed link flow data. Bertsimas et al. (2015) considered the inverse variational inequality problems to estimate the congestion function on a road network from traffic count data. Wang et al. (2016) proposed a two-stage algorithm to simultaneously estimate origin-destination matrices and link choice proportions by incorporating a dynamic dispersion parameter into the route choice model. They presented a model calibration procedure that enables the simultaneous estimation of the dispersion parameter, link choice proportions, and OD matrix. Hong et al. (2017) employed inverse optimization to calibrate the joint probability density function of taste parameters for best signifying the user optimality of observed routes and tested the predictability on an intensive trip dataset obtained from Seoul metropolitan area. Guarda and Qian (2023) extended classical bi-level formulations to estimate the travelers’ route choice preferences with multiple attributes using system-level data. By incorporating users’ decision-making equilibrium into preference appraisal, the dynamic estimation process may account for a more realistic situation and provide a smart “estimate, then equilibrate” scheme.

This study proposes an “inverse optimization” method to estimate the commuters’ schedule preference and crowding perception in an urban rail corridor system. The inverse optimization model is formulated as a bi-level programming problem, where the lower-level is a many-to-one rail corridor commuting equilibrium model with preference parameters to be estimated and the upper-level is the parameter optimization problem. To measure the smart card holders’ travel choice preferences from massive rail transit data, it is of utmost importance to consider commuters’ heterogeneity in travel time preferences, i.e., desired arrival time and choice set of departure times or arrival times. Therefore, we formulate a logit-based stochastic user equilibrium (SUE) model embracing multi-class commuters with different travel time preferences to approximate the observed user optimality, the equilibrium condition of which adheres to the multinomial logit-based departure (arrival) time choice principle. Furthermore, a sensitivity analysis-based (SAB) solution framework is proposed to solve the bi-level programming model of inverse optimization and then is applied to an empirical study on the Beijing Subway Batong Line to validate the robustness and practicality of the proposed method.

The remainder of the paper is organized as follows. Section 2 introduces the basic consideration of the urban rail commuting model and inverse optimization-based preference estimation method. Section 3 presents the solution method framework for inverse optimization. Section 4 describes the dataset used and explains how the observed rail trip data has been normalized for estimation. Section 5 starts with discrete choice estimation and then proceeds to demonstrate the implementation of the inverse optimization method through an empirical study on the Beijing Subway Batong Line, followed by discussions of the results. Finally, Section 6 concludes the paper and provides some remarks for further research.

2 Commuting Model and Inverse Optimization

In this section, we start with introducing individuals’ generalized travel cost function for rail commuting. Subsequently, a logit-based SUE rail commuting model, accommodating heterogeneous commuters, is formulated to capture the morning commuting patterns for a many-to-one urban rail corridor system. Based on the observed commuting choices and the formulated equilibrium model, the inverse optimization method is further proposed for preference estimation.
## 2.1 Generalized rail commuting cost

Rail transit commuters usually determine their departure time based on several main factors such as travel time/delay, schedule delay, transit fare and in-vehicle crowding during morning peak period (Huang et al., 2005, 2007; Tian et al., 2007a, 2007b, 2009, 2021; Wu and Huang, 2014; de Palma et al., 2015, 2017). We consider that all rail commuters reside and work in proximity to the stations, and the travel processes from home to the station and from the station to the workplace remain relatively fixed, generating a constant cost. As a result, the travel processes at both ends do not influence their choice of departure time. The individual generalized commuting cost for rail passengers, who enter the station at $t_d$, board the train at $t_i$, alight from the train at $t_o$, and exit the station at $t_a$ with a desired arrival time $t^*$ at the workplace can be formulated as:

$$ C(t_d, t_i, t_o, t_a, t^*) = F(t_d) + \alpha T(t_d, t_a) + SDC(t_a, t^*) + G(t_i, t_o), \quad (1) $$

where $F(t_d)$ represents the transit fare departing at $t_d$. The second term on the right-hand side of Eq. (1) corresponds to the travel time cost associated with rail commuting, $\alpha$ is the marginal time utility cost of trip time, and $T(t_d, t_a) = t_d - t_a$ is the total travel time. The third term, denoted as $SDC(t_a, t^*)$, represents the schedule delay cost incurred whenever arriving either early (before $t^*$) or late (after $t^*$) at the workplace, and is defined as the summation of schedule delay early and late costs as follows:

$$ SDC(t_a, t^*) = \beta T_E(t_a, t^*) + \gamma T_L(t_a, t^*), \quad (2) $$

where $T_E(t_a, t^*) = \max (t^* - t_a, 0)$ is the early arrival time, and $T_L(t_a, t^*) = \max (t_a - t^*, 0)$ is the late arrival time. $\beta$ and $\gamma$ represent the marginal time utility cost for early arrivals and late arrivals at the workplace, respectively.

The fourth term, denoted as $G(t_i, t_o)$ on the right-hand side of Eq. (1), accounts for the total crowding cost when boarding a train at $t_i$ and alighting at $t_o$, and is assumed to be the integral of the crowding disutility per unit time over the in-vehicle time:

$$ G(t_i, t_o) = \int_{t_i}^{t_o} g \cdot f(n(t)) dt, \quad (3) $$

where $g$ is a non-negative parameter that represents the individual’s in-vehicle crowding perception preference or value of the in-vehicle crowding per unit time, and $f(n(t))$ signifies the in-vehicle crowding degree. $f(n(t))$ is assumed to be an increasing function of $n(t)$, where $n(t)$ is the number of passengers in the vehicle at instant $t$.

### 2.2 Rail corridor commuting equilibrium model

![Figure 1: Rail transit commuting corridor with multiple origins and a single destination.](image)

As shown in Figure 1, consider a rail transit commuting corridor with multiple origins/stations $O_i, i \in \zeta = \{1, 2, \ldots, I\}$ in the sequence of decreasing distance to a single destination $D$. Let $J = \{1, 2, \ldots\}$ denote
the set of successive train services departing from the farthest station \( O_1 \) to \( D \) during the morning peak period, and \( t_\zeta, i \in \zeta = \{1, 2, \ldots, I\} \) denote the train travel time from \( O_i \) to \( O_{i+1} \) including the stop time at \( O_i \). Commuters are assumed to be heterogenous in travel time preferences, i.e., desired arrival time and choice set of departure times or arrival times, but homogeneous in others (Note that all smart card-holders’ socioeconomic attributes are unidentified). Therefore, the commuters from origin \( O_i \) can be divided into multiple classes, a specific one of which is denoted as \( k, k \in \mathcal{K}_i = \{1, 2, \ldots\} \) with identical desired arrival time \( t_{i,k} \) and departure time choice set or train choice set \( J_{i,k} \), and \( N^k_i \) denotes the total number of class \( k \) commuters departing from station \( O_i \).

The transit fare is assumed to remain constant across different train services between the given origin \( O_i \) and destination \( D \) (reflecting common practice in China). Therefore, it is not expected to influence commuters’ departure time choices. This assumption can be relaxed to accommodate a time-dependent fare case in future research. According to Eq. (1), the generalized cost experienced by class \( k \) commuters taking train \( j \) from station \( O_i \) becomes:

\[
C_{i,j}^k = \alpha T(i, j) + \beta T_\zeta^k(i, j) + \gamma T_\zeta^k(i, j) + g \int_{t_{i,i(j)}}^{t_{i,i(j)}} f(n(t)) dt, \forall i \in \zeta, k \in \mathcal{K}_i, j \in J_{i,k},
\]

where \( T(i, j) \) is the trip time of taking train \( j \) from entering station \( O_i \) to exiting station \( D \), \( T_\zeta^k(j) = \max(t_{i,k} - t_{d(j)}), 0) \) and \( T_\zeta^k = \max(t_{d(j)} - t_{i,k}, 0) \) are schedule delay time arriving early and schedule delay time arriving late of taking train \( j \), and \( t_{d(j)} \) is the arrival time at exit station \( D \) of taking train \( j \). Furthermore, \( t(i, j) \) is the boarding time of taking train \( j \) at station \( O_i \), and \( t_{o}(j) \) is the alighting time at station \( D \) of taking train \( j \) and the in-vehicle crowding cost term can be formulated as:

\[
g \int_{t_{i,i(j)}}^{t_{i,i(j)}} f(n(t)) dt = g \sum_{m=i}^{t} \sum_{l=1}^{m} \sum_{p=1}^{n_{i,j}} \delta_{l,p}^i \epsilon_m, \forall i \in \zeta, k \in \mathcal{K}_i, j \in J_{i,k},
\]

where \( \sum_{p=1}^{n_{i,j}} \sum_{l=1}^{m} \delta_{l,p}^i \epsilon_m \) is the total number of commuters in train \( j \) between station \( O_m \) and station \( O_{m+1} \), \( n_{i,j}^p \) is the number of class \( p \) commuters taking train \( j \) from station \( O_i \), and \( \delta_{l,p}^i \) is the relationship between the class-specific train choice set and train services. Specifically, \( \delta_{l,p}^i \) equals 1 when \( j \in J_{i,p} \), otherwise \( \delta_{l,p}^i \) equals 0.

Suppose all commuters aim to minimize their generalized travel costs when choosing departure times (or train services) and thus balance among various factors such as travel time, schedule delay, and in-vehicle crowding. However, commuters could not acquire the perfect information on traffic status for different departure times, so they would take trains according to their perceived travel costs \( \tilde{C}_{i,j}^k \):

\[
\tilde{C}_{i,j}^k = C_{i,j}^k + \epsilon_{i,j}^k, \forall i \in \zeta, k \in \mathcal{K}_i, j \in J_{i,k},
\]

where \( \epsilon_{i,j}^k \) is a random term of differences between the deterministic travel cost and individual perceived travel cost. At equilibrium, commuters departing from the same station and belonging to the same class should have an identical and minimum perceived travel cost regardless of the trains they choose, and no one could reduce his or her perceived travel cost by unilaterally changing his/her choice of train service. Hence, the probability of class \( k \) commuters from station \( O_i \) choosing train \( j \) can be expressed as:

\[
P_{i,j}^k = \Pr(\tilde{C}_{i,j}^k \leq \tilde{C}_{i,s}^k), \forall i \in \zeta, k \in \mathcal{K}_i, j \in J_{i,k}; \forall s \in J_{i,k}, s \neq j.
\]

The perceived travel costs \( \tilde{C}_{i,j}^k \) can be defined as the corresponding travel disutility. According to
the random utility theory, if the random terms $\epsilon^k_{i,j}$ are independent and identically Gumbel-distributed, then the probability of class $k$ commuters choosing train $j$ from station $O_i$ can be expressed by the multinomial logit formula\textsuperscript{1}:

$$p^k_{i,j} = \frac{\exp(-\theta c^k_{i,j})}{\sum_{s \in J_{i,k}} \exp(-\theta c^k_{s,j})}, \forall i \in \zeta, k \in K_i, j \in J_{i,k},$$

where $\theta$ is a non-negative scale parameter related to the variance of $\epsilon^k_{i,j}$. At the state of SUE, the number of class $k$ commuters taking train $j$ from station $O_i$ can be written as:

$$n^k_{i,j} = N^k_i p^k_{i,j}, \forall i \in \zeta, k \in K_i, j \in J_{i,k}.$$  

With a given train timetable, we could formulate the following mathematical programming problem for which the resulting complementary slackness conditions give rise to a logit-based SUE departure rate/boarding flow distribution $n = \{ n^k_{i,j} | i \in \zeta, k \in K_i, j \in J_{i,k} \}$:

$$\min \limits_n \; Z(n) = \frac{1}{\theta} \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} n^k_{i,j} \ln n^k_{i,j} + \sum_{m=1}^{l} \left( \sum_{j \in J} \sum_{k \in K_i} n^k_{i,j} f(v) dv \right) \tau_m$$

$$+ \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} n^k_{i,j} \left( \alpha T(i,j) + \beta T^k(i,j) + \gamma T^k(i,j) \right).$$

s.t. $$\sum_{j \in J_{i,k}} n^k_{i,j} = N^k_i, \forall i \in \zeta, k \in K_i,$$

$$n^k_{i,j} \geq 0, \forall i \in \zeta, k \in K_i, j \in J_{i,k}.$$  

The objective function (10a) is a derivative version of Fisk’s SUE model that results in a logit-based traffic assignment (Fisk, 1980). Eqs. (10b) and (10c) are conservation constraints and non-negative constraints for departure rates/boarding flows, respectively. Let $\omega$ denote the predefined preference parameter set $(\alpha, \beta, \gamma, g, \theta)$. As the first term of the objective function is strictly convex on $n^k_{i,j}$, and the remaining is convex conditional on the non-negativity of $\omega$, coupled with the linearity of constraints in the feasible region, the above mathematical problem is a convex minimization problem with a unique solution (see the proof in Appendix A). Let $n^*(\omega) = \{ n^k_{i,j}^*(\omega) | i \in \zeta, k \in K_i, j \in J_{i,k} \}$ be the equilibrium departure rates at a given $\omega$. The following first-order optimality conditions of Eqs. (10) hold at the equilibrium state $n^*(\omega)$:

\textsuperscript{1} When concentrating on specific and observable preference ingredients/components derived from commuters’ smart card records, it is essential to acknowledge the inherent limitation associated with the assumption of independence from irrelevant alternatives (IIA) in the logit formula. Although accommodating commuters’ heterogeneity in travel time preferences, the challenge remains in capturing all sources of correlation among train alternatives. Moreover, the omission of the travel processes at both ends of the rail commute also raises concerns about the IIA assumption (As detailed in Section 5.1, our empirical tests on discrete choice estimation reveal the difficulty of ensuring the IIA assumption when solely relying on smart card data). While the multinomial logit model grapples with empirical challenges in departure time choice, the logit-based SUE presented in Eq. (10) serves to simulate the group equilibrium states and implement the mechanisms of inverse optimization through the measurements of system-level aggregations. A closed-form expression for logit-based equilibrium strikes a balance between behavioral realism and mathematical tractability (Guarda and Qian, 2023).
\begin{align}
n_{i,j}^{k*}(\omega) & \left( \frac{1}{\theta} \left( \ln n_{i,j}^{k*}(\omega) + 1 \right) + C_{i,j}^k \left( n^{*}(\omega), \omega \right) - \mu_k^i \right) = 0, \quad \forall i \in \zeta, k \in K_i, j \in J_{i,k}, \quad (11a) \\
\frac{1}{\theta} \left( \ln n_{i,j}^{k*}(\omega) + 1 \right) + C_{i,j}^k \left( n^{*}(\omega), \omega \right) - \mu_k^i & \geq 0, \quad \forall i \in \zeta, k \in K_i, j \in J_{i,k}, \quad (11b) \\
\sum_{j \in J_{i,k}} n_{i,j}^{k*}(\omega) & = N_k^i, \quad \forall i \in \zeta, k \in K_i, \quad (11c) \\
n_{i,j}^{k*}(\omega) & \geq 0, \quad \forall i \in \zeta, k \in K_i, j \in J_{i,k}, \quad (11d)
\end{align}

where $C_{i,j}^k(n^*(\omega), \omega)$ is the generalized cost from Eq. (4) with given $\omega$ and $n^*(\omega)$, $\mu_k^i$ is the Lagrange multiplier associated with constraint (10b). When $n_{i,j}^{k*}(\omega) > 0$, from Eqs. (11a) and (11c) we could derive the equivalent train-dependent choice probability governed by the logit formula (8).

### 2.3 Parameters estimation through inverse optimization

A typical optimization problem is a forward problem of identifying the values of observable parameters (an optimal solution), given the values of model parameters in the objectives and constraints. An inverse optimization, in contrast, involves inferring the values of model parameters in the objectives or constraints, so that a set of prescribed solutions (observations) become optimal (Ahuja and Orlin, 2001; Chan et al., 2023).

The legitimacy of employing inverse optimization to estimate preference parameters is based on the fundamental assumption that observed choice behavior is a manifestation of user optimality. Extending the same intention to our work is to derive a set of preference parameters revealing commuters’ schedule preference and crowding perception so that the solution of rail corridor commuting equilibrium closely approximates the observed group commuting pattern obtained through the smart card data. Utilizing individuals’ travel records over weekdays, desired arrival times and train choice sets for multiple commuter classes can be identified. This involves a clustering analysis of temporal characteristics of individuals’ commuting patterns as detailed in Section 4.1. The user class-specific schedule delay time for each train service can thus be calculated. With a specific set of preference parameters, the departure time choices of multi-class commuters and the in-vehicle crowding costs for different train services are endogenously determined upon reaching a departure time choice equilibrium. Therefore, the simultaneity effects between in-vehicle crowding and departure time choices can be addressed, and the aggregated choice observations can be approximated by calibrating the preference parameters in the equilibrium model, thereby revealing the dynamic trade-offs among various cost components hidden beneath the static choice results.

The inverse optimization method can be formulated as a bi-level programming problem, where the lower-level (L) is the rail corridor commuting equilibrium model with preference parameters to be estimated, and the upper-level (U) is the parameter optimization problem. By solving the lower-level model, the endogenous commuting equilibrium pattern of each commuter class at different stations can be attained, given a specific set of preference parameters $\omega$. The objective of the upper-level model is to minimize the squared sum of the difference between the equilibrium departure rates and observed departure rates obtained from real-world mobility data. In accordance with the assumption of rational behavior, the preference parameters are constrained to be non-negative values. It follows that
where $\hat{n}_{i,j}^k(\omega)$ is the equilibrium departure rate under given parameters $\omega$ and $\hat{n}_{i,j}^k$ is the observed departure rate/boarding flow of class $k$ commuters taking train $j$ from station $O_i$. The lower-level commuting equilibrium state is determined by four utility preference parameters along with a scale parameter. Consequently, there are five parameters of $\omega$, i.e. $\omega = (\alpha, \beta, \gamma, g, \theta)$, to be estimated. To solve the above bi-level programming model, it is essential to account for the impact of the subtle adjustment in the estimated preference parameters on the equilibrated departure rates. As the interaction between the upper-level and lower-level models is characterized by a nonlinear and implicit function form, bi-level programming typically represents a non-convex optimization problem which has been proven to be NP-hard and challenging to solve through an exact algorithm (Ben-Ayed and Blair, 1990). Even though all functions and sets involved in upper-level inverse optimization and lower-level forward optimization are convex, solving the inverse optimization problem remains NP-hard (Aswani et al., 2018). In this study, a convergent line search solution method based on sensitivity analysis of the lower-level model is designed for the inverse optimization. By assessing the sensitivity of the lower-level model to variations in the estimated preference parameters, one can iteratively refine these preference parameters and ultimately achieve a more accurate approximation of the actual commuting pattern.

### 3 Solution Method Framework

In the lower-level commuting equilibrium model (10), the preference parameters of $\omega$ only appear in separate terms of the objective function and thus different parameters setting may yield an equivalent equilibrium state. To obtain a unique estimation from the inverse optimization model, we reset the preference parameters as $\bar{\omega} = (\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{g})$ to ensure a certain equilibrium state corresponding to a unique preference parameter setting, where $\bar{\alpha} = \theta \alpha$, $\bar{\beta} = \theta \beta$, $\bar{\gamma} = \theta \gamma$ and $\bar{g} = \theta g$, and thus Eq. (10a) becomes

$$
\min_{n} \bar{Z}(n, \bar{\omega}) = \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} n_{i,j}^k \ln n_{i,j}^k + \sum_{m=1}^{t} \left( \sum_{j \in J} \sum_{l \in K_j} \sum_{p \in J_m} n_{l,pj}^m f'(\nu) \nu \right) \tau_m
$$

$$
+ \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} n_{i,j}^k \left( \bar{\alpha} T(i,j) + \bar{\beta} T_E(i,j) + \bar{\gamma} T_L(i,j) \right)
$$

and the upper-level problem of Eq. (12) becomes the following:

$$
\min_{\omega} \bar{F}(n^*(\bar{\omega})) = \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} \left( n_{i,j}^{k*}(\bar{\omega}) - \hat{n}_{i,j}^k \right)^2
$$

\[ s.t. \quad \bar{\omega} \geq 0 \]
where the explicit expressions of the derivatives of equilibrated departure rates $n(\omega)$ with respect to preference parameters $\omega$ can be derived through sensitivity analysis of the lower-level rail corridor commuting equilibrium model.

3.1 Sensitivity analysis for lower-level equilibrium model

Sensitivity analysis-based (SAB) methods have been extensively applied in various behavioral traffic equilibrium analysis, especially in the development of solution methods for continuous network design problems (Wang et al., 2022; Yang and Bell, 2007) as well as traffic control problems with user optimality (Rey and Levin, 2019; Yang, 1995; Yang and Yagar, 1995). Here we employ the sensitivity analysis method to compute the derivatives of equilibrated departure rates $n(\omega)$ with respect to preference parameters $\omega$ in the lower-level rail corridor commuting equilibrium model. The general methods and proof for sensitivity analysis of the nonlinear programming problem can be found in the related literature (e.g., Fiacco, 1983).

Let $n^*(\omega) = (n^*_{i,j}(\omega), i \in \zeta, k \in K, j \in J_{i,k})^T$ denote the equilibrated departure rate/boarding flow distribution derived from the lower-level commuting equilibrium model of Eqs. (13), (10b) and (10c). Note that $n^*(\omega) > 0$ because the logit-based stochastic flow assignment makes each class of commuters choose every train service with a specified probability of Eq. (8). Thereby, without assuming the strict complementary slackness condition when $n^*_{i,j} = 0$, the Kuhn-Tucker conditions of the lower-level commuting equilibrium model reduce to:

$$C(n^*(\omega), \omega) - \Delta^T \cdot \mu = 0,$$

$$\Delta \cdot n^*(\omega) - N = 0,$$

where $C(n^*(\omega), \omega) = (\ln n^*_{i,j}(\omega) + 1 + C^k_{i,j}(n^*(\omega), \omega), i \in \zeta, k \in K, j \in J_{i,k})^T$, $\mu = (\mu^i, i \in \zeta, k \in K)^T$, $N = (N^i, i \in \zeta, k \in K)^T$, and $\Delta$ is the incidence matrix representing the relationship between the total number of class-specific commuter and train-dependent departure rates of multiple commuter classes.

Taking the derivatives on both sides of Eqs. (15) with respect to $\omega$, we have:

$$\begin{bmatrix} \nabla_{\omega} n^*(\omega) \\ \nabla_{\omega} \mu^*(\omega) \end{bmatrix} = \begin{bmatrix} \nabla_{\omega} C(n^*(\omega), \omega) & -\Delta^T \\ \Delta & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\nabla_{\omega} C(n^*(\omega), \omega) \\ 0 \end{bmatrix}.$$

Assume

$$\begin{bmatrix} \nabla_{\omega} C(n^*(\omega), \omega) & -\Delta^T \\ \Delta & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}.$$

It can be obtained that:

$$\mathbf{B}_{11} = \nabla_{\omega} C(n^*(\omega), \omega) \nabla_{\omega} C(n^*(\omega), \omega)^{-1} - \nabla_{\omega} C(n^*(\omega), \omega)^{-1} \Delta^T \left( \nabla_{\omega} C(n^*(\omega), \omega)^{-1} \Delta^T \right)^{-1} \Delta \nabla_{\omega} C(n^*(\omega), \omega)^{-1}.$$

Therefore,

$$\nabla_{\omega} n^*(\omega) = -\mathbf{B}_{11} \nabla_{\omega} C(n^*(\omega), \omega).$$
Eq. (19) is the explicit expression of the derivatives of equilibrated departure rates \( n^* (\vec{o}) \) with respect to preference parameters \( \vec{o} \).

### 3.2 SAB solution method for inverse optimization

The SAB solution method is a line search algorithm for a bi-level programming problem, which linearizes the implicit relationship between upper-level decision variables and lower-level decision variables based on the sensitivity analysis of the lower-level problem. From the sensitivity analysis of the rail corridor commuting equilibrium model, the first-order Taylor approximation of the equilibrated departure rates \( n^* (\vec{o}) \) with respect to the preference parameters \( \vec{o} \) at \( \vec{o}^p \) can be given as:

\[
n^* (\vec{o}) \approx n^* (\vec{o}^p) + \nabla n^* (\vec{o}^p) \cdot (\vec{o} - \vec{o}^p),
\]

where the derivative information of \( \nabla n^* (\vec{o}^p) \) can be obtained from Eq. (19).

By substituting Eq. (20) into the objective function of the upper-level problem, the upper-level parameter optimization problem is transformed into a convex quadratic optimization problem in the vicinity of \( \vec{o}^p \). This transformation enables the problem to be resolved using a range of efficient solution algorithms, e.g., the active-set algorithm, or employing commercial solvers such as Gurobi. The optimal solution obtained from the upper-level problem is then used to update the preference parameters in the lower-level commuting equilibrium model. After solving the lower-level problem again, the first-order Taylor approximation of the equilibrated departure rates with respect to the updated preference parameters is computed. By iterating this loop, the solution process of bi-level programming is segmented into a sequence of convex optimizations, effectively mitigating the challenges posed by non-convexity and reducing computational complexity. Since the linearity of Eq. (20) accurately approximates the equilibrated departure rates \( n^* (\vec{o}) \) only within a restricted neighborhood of \( \vec{o}^p \), and a large step size may lead to the algorithm diverging or becoming entirely ineffective, we set a fixed neighborhood radius \( \delta \) at \( \vec{o}^p \) in the algorithm to ensure a high degree of accuracy for approximation. The detailed procedure of the solution method is as follows:

**Step 1 Initialization:** Set an arbitrary initial feasible parameter value \( \vec{o}^0 \), and set \( p \leftarrow 0 \);

**Step 2 Solve the lower-level problem:** Solve the lower-level rail corridor commuting equilibrium model with the given parameter value \( \vec{o}^p \) to obtain \( n^* (\vec{o}^p) \);

**Step 3 Sensitivity analysis:** Compute the derivative of \( n \) with respect to \( \vec{o}^p \) according to the sensitivity analysis method;

**Step 4 Solve the upper-level problem:** Substitute Eq. (20) into the upper-level model. Solve the upper-level model satisfied with restricting conditions \( |\vec{o} - \vec{o}^p| \leq \delta \) and get \( \vec{o}^{p+1} \), where \( \delta \) is a fixed neighborhood radius;

**Step 5 Termination:** If \( \max |\vec{o}^{p+1} - \vec{o}^p| \leq \delta^* \), where \( \delta^* \) is a relatively small value and \( \delta^* < \delta \), then terminate and return \( \vec{o}^{p+1} \) as \( \vec{o}^* \); otherwise, set \( p \leftarrow p + 1 \), and go to step 2.

where in Step 2, the lower-level rail corridor commuting equilibrium model is a convex programming problem with linear constraints and can be solved through the convex combination algorithm discussed in the next section.

The upper-level objective function \( \tilde{F}(n^* (\vec{o})) \) is uniformly continuous with respect to \( n^* (\vec{o}) \). Given that \( n^* (\vec{o}) \) is a vector of Lipschitz continuous functions of \( \vec{o} \) (Yen, 1995), \( \tilde{F}(n^* (\vec{o})) \) also exhibits uniform continuity in \( \vec{o} \). Note that as \( \vec{o} \) is bounded (the preference parameters are usually
within a reasonable range), \( \bar{F}(n^*(\bar{\omega})) \) is bounded as well. The proof of convergence for the iterative estimation procedure can be found in Appendix B. By comparison, we also present a refined solution method employing the globally convergent norm-relaxed method of feasible direction (NRMFD) algorithm (Cawood and Kostreva, 1994; Wang et al., 2021), specifically applied to solving the bi-level programming of continuous network design problem. Using the dataset referenced in Section 5.2.2, both algorithms can converge to nearly identical parameter estimation results. However, the NRMFD exhibits a slower convergence in the subsequent iterations when fine-tuning the solution, suggesting a need for integration with more efficient approaches to enhance the convergence (Wang et al., 2022).

### 3.3 Solution method for lower-level equilibrium model

The main idea of solving the rail corridor commuting equilibrium model is based on the convex combination iteration. That is, in each iteration, firstly we find a descent direction of the objective function and determine an optimal step size, and then obtain the starting point for the next iteration by intercepting an optimal step size in the descent direction. This iterative process continues until convergence to the optimal solution. The detailed procedure of the solution method is as follows:

**Step 1 Initialization:** Calculate the generalized travel cost \( C_{i,j}^k(n^0, \bar{\omega}), \forall i \in \zeta, k \in K, j \in J_{i,k} \) on the basis of zero flow \( n^0 = (n_{i,j}^0, i \in \zeta, k \in K, j \in J_{i,k})^T = 0 \). Then assign the multi-class commuter flow according to Eq. (21) to obtain the initial flow \( n^q = (n_{i,j}^q, i \in \zeta, k \in K, j \in J_{i,k})^T \), and set \( q = 1 \).

\[
n_{i,j}^k = N_i^k \frac{\exp \left( -C_{i,j}^k(n^0, \bar{\omega}) \right)}{\sum_{s \in J_{i,k}} \exp \left( -C_{i,s}^k(n^0, \bar{\omega}) \right)}, \quad \forall i \in \zeta, k \in K, j \in J_{i,k}.
\]

**Step 2 Direction:** Calculate the generalized travel cost \( C_{i,j}^k(n^q, \bar{\omega}), \forall i \in \zeta, k \in K, j \in J_{i,k} \) on the basis of \( n^q \), and then assign the multi-class commuter flow according to Eq. (22) to obtain flow \( \bar{n}^q = (\bar{n}_{i,j}^q, i \in \zeta, k \in K, j \in J_{i,k})^T \).

\[
\bar{n}_{i,j}^k = N_i^k \frac{\exp \left( -C_{i,j}^k(n^q, \bar{\omega}) \right)}{\sum_{s \in J_{i,k}} \exp \left( -C_{i,s}^k(n^q, \bar{\omega}) \right)}, \quad \forall i \in \zeta, k \in K, j \in J_{i,k}.
\]

**Step 3 Step size:** Set \( 0 \leq \lambda \leq 1 \) and \( n^{q+1} = n^q + \lambda(\bar{n}^q - n^q) \). Use the bisection method to determine \( \lambda^* \) that satisfies the following equation:

\[
\left( \bar{n}^q - n^q \right)^T \cdot C(n^{q+1}) = 0,
\]

where \( C(n^{q+1}) = (\ln h_{i,j}^k + 1 + C_{i,j}^k(n^{q+1}, \bar{\omega}), i \in \zeta, k \in K, j \in J_{i,k}) \).

**Step 4 Flow update:** Set \( n^{q+1} = n^q + \lambda^*(\bar{n}^q - n^q) \).

**Step 5 Termination:** if

\[
\frac{\sqrt{(n^{q+1} - n^q)^T (n^{q+1} - n^q)}}{n^{q+1} \cdot n^q} < \eta,
\]
then terminate and return $n^{q+1}$, where $\eta$ is the pre-set error limit; otherwise, set $q \leftarrow q+1$, and go to Step 2.

Following Huang (1995), we present a proof that the direction $(\vec{n}^q - n^q)$ determined in Step 2 can decrease the objective function value of Eq. (13) in Appendix C. In addition, in the process of determining the optimal step size in the descent direction $(\vec{n}^q - n^q)$ using the bisection method, Eq. (23) is derived from the one-dimensional extremum problem of minimizing the value of $\tilde{Z}(n^{q+1}, \vec{o})$ as stated in Eq. (13). In each iteration, the flow pattern is updated according to Eq. (24). Subsequently, the in-vehicle crowding cost is also updated in response to the changes in the flow pattern. Eq. (24) characterizes the deferring or advancing behavior of commuters’ departure time choices in response to lagged effects of travel costs and behavioral inertia. This dynamic adjustment process prompts commuters to switch to train services of lower generalized cost through balancing travel time cost, schedule delay cost and in-vehicle crowding cost, which is similar to the day-to-day choice evolutionary mechanism (e.g., Guo et al., 2017). The solution procedure of SUE endogenizes commuters’ departure time choices and ultimately converges to the pre-defined user optimality driven by specified utility preference parameters.

4 Data

![Figure 2: Average daily number of commuters departing along the rail corridor system along the Beijing Subway Batong Line.](image)

The dataset used for inverse optimization to calibrate urban rail commuters’ utility preference are derived from the subway trip data of the smart card system over the 15 weekdays, from October 9, 2017 to October 29, 2017, along the Beijing Subway Batong Line, which contains the information of passengers’ smart card number, location and time of origin station (tap-in) and destination station (tap-out). The Batong Line is an 18.065 km long subway line that spans from Tongzhou District to the Beijing Central Business District. It serves as a crucial urban rail corridor, connecting residential areas in the eastern part of Beijing to the bustling commercial area. During the morning peak period, many rail commuters residing along the subway Batong Line depart to their workplaces near Sihui Dong station, and thus the subway Batong Line can be modeled as a many-to-one urban rail corridor system.
By sifting through the trip data of passengers who commute at least 9 times during the morning peak period (AM 7:00~10:00) on 15 weekdays, namely more than 3 times a week on average, Figure 2 shows the average daily number of commuters departing from multiple origins to Sihui Dong station over 15 weekdays. Due to the relatively intensive commuters gathering at the stations of Tu Qiao, Liyuan, Guoyuan and Tongzhou Beiyuan, we take these 4 stations as the main origins of the many-to-one rail corridor system for illustration, which are indexed by \(i = 1, 2, 3, 4\), respectively. From these stations, a total of 30,079 valid trip data for 2,408 commuters are obtained, which are used to analyze their commuting patterns and travel preferences.

### 4.1 Commuter classification and travel time preferences

Given the obvious preference heterogeneity in desired arrival times and timing of departure time choices among various commuters, this paper explores a many-to-one urban rail corridor system catering to multi-class commuters with diverse travel time preferences. However, the information about commuters’ travel time preferences, specifically, their desired arrival time and choice set of departure times or arrival times, are not directly revealed in their trip data. In order to infer the value of individuals’ desired arrival time \(t_{ik}^*\), we refer to the disaggregate analysis method developed by Cats and Ferranti (2022) to identify the habitual temporal travel patterns of smart card-holders and determine their underlying travel time preferences.

The calculation of the number of trips a commuter makes within a specific temporal window over a period is conducted to examine their travel time preferences during the morning peak period. We take 10 minutes as the granularity to aggregate commuters’ trip data and discretize the morning peak period from 7:00 AM to 10:00 AM into 18 intervals by defining \(\text{hour}(t) \in \{7, 8, 9\}\) and \(\text{minute}(t) \in \{10, 20, 30, 40, 50, 60\}\). After counting the arrival time frequency \(f_n(t)\) that commuter \(n\) performs over 15 weekdays for every 10 minutes from 7:00 am to 10:00 am, we have the resulting vectors with 18 entries as the commuting pattern of commuter \(n\):

\[
P_n = \left[ \sum_{\text{hour}(t) = 7} f_n(t), \sum_{\text{hour}(t) = 7} f_n(t), \ldots, \sum_{\text{minute}(t) = 60} f_n(t) \right].
\]  

(26)

Figure 3: Silhouette index value for K-means clustering approach.
While commuters may differ in their commuting patterns $P_n$, it is expected that commuters of a certain group exhibit similar travel preferences. Consequently, the commuters are classified into multiple groups through the application of the K-means clustering approach to their commuting patterns. In the case of commuters originating from multiple locations such as Tu Qiao, Liyuan, Guoyuan, and Tongzhou Beiyuan, the numbers of clusters are chosen according to the silhouette index, an indicator used for evaluating the goodness of the clustering and a higher value suggesting that cluster members exhibit similarity amongst each other whilst being discernible from other clusters (Rousseeuw, 1987). As shown in Figure 3, to ensure a sufficient quantity of observed departure rate/boarding flow for each commuter class, the numbers of clusters $|K|$ are set in line with the maximum or local maximum of silhouette index, that is, 5, 6, 5 and 6 for commuters from stations of Tu Qiao, Liyuan, Guoyuan and Tongzhou Beiyuan, respectively.

![Figure 4: Arrival time features of multi-class commuters from multiple stations.](image)

We take the cluster centers as the arrival time feature vectors for each respective class of commuters. As shown in Figure 4, these arrival time features represent the habitual temporal travel patterns of each commuter class. Given that each class of commuters exhibits common travel time preferences according to their arrival time features, the arrival time corresponding to the peak value of each feature vector, which suggests the highest frequency of arrival time, is considered as the desired arrival time of each commuter class. Furthermore, the period with arrival time frequency greater than

---

2 Using the most frequently observed arrival time as an approximate proxy for the desired arrival time is a heuristic conjecture due to limited data access. When assuming homogeneity among commuters and constant in-vehicle travel time, the desired arrival times coincide with the most frequent arrival time for both the single origin and destination (Appendix D) and many-to-one rail transit system (Tian et al., 2007a), as confirmed by user equilibrium-based theoretical analysis. However, considering the heterogeneity of users, the randomness of choices and the variation in travel time across different trains, the highest frequency of arrivals may not fully capture individuals’ true preferences. Future research could benefit from the improved identification of desired arrival times by incorporating more authentic data sources, such as those collected through stated preference surveys.
zero is regarded as the range of arrival time choices for each commuter class. Taking the cluster center of the first commuter class at Tu Qiao station as an example, the desired arrival time is determined to be 8:35am, which corresponds to the peak value of the arrival time feature, and the time range of arrival time choices is identified as 8:00am~9:10am.

4.2 Data processing and variable measurement

As shown in Figure 5, there are 46 train services available for commuters at stations of Tu Qiao, Liyuan, Guoyuan and Tongzhou Beiyuan during the morning peak period (AM 7:00~10:00) according to train timetable data. Assuming that each commuter walks directly to the automatic fare gate after alighting from the train without any pause on the platform, the morning peak period can be divided into 46 intervals. These intervals are determined based on the train arrival time at Sihui Dong station, along with the time required for commuters to walk from the platform to the automatic fare gate. As a result, the commuters arriving at each interval can be matched with a certain train service.

![Figure 5: Train services during the morning peak period.](image)

To obtain the observed user optimality within a day, the measurement of some train choice-related variables is derived from historical averages over 15 weekdays. Let \( t_j, j \in J = \{1,2,...,46\} \) denote the arrival time interval of each train service. The average trip time \( T(i, j) \), as well as the average arrival time \( A_t(j) \) of taking corresponding train services, can thus be obtained by measuring commuters’ trip data of each arrival time interval over 15 weekdays, which are regarded as the trip time and the arrival time of taking each train service. In addition, the passenger flow distribution of each train service is also calculated through historical average trip data. The specific measurements of train choice-related variables derived from synthesized smart card data and train timetable data are summarized in Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T(i, j) )</td>
<td>Trip time of taking train service ( j ) from station ( O_i ), i.e., seconds of the average trip time of commuters from station ( O_i ) arriving at time interval ( t_j ).</td>
</tr>
<tr>
<td>( t_a(j) )</td>
<td>Arrival time of taking train service ( j ), i.e., the average arrival time of commuters arriving at time interval ( t_j ).</td>
</tr>
</tbody>
</table>
\( t_{i,k}^* \) Desired arrival time of class \( k \) commuters from station \( O_i \), i.e., the arrival time corresponding to the peak value of the arrival time feature vectors for class \( k \) commuters.

\( J_{i,k} \) Train choice set of class \( k \) commuters from station \( O_i \), i.e., train services available during the time range of arrival time choices for class \( k \) commuters.

\( T_{E}^{k}(i,j) \) Schedule delay time arriving early, in seconds, of class \( k \) commuters from station \( O_i \) arriving at time interval \( t_j \), i.e., \( T_{E}^{k}(t_{i,k}) = \max(t_{i,k}^* - t_{i}(j), 0) \).

\( T_{L}^{k}(i,j) \) Schedule delay time arriving late, in seconds, of class \( k \) commuters from station \( O_i \) arriving at time interval \( t_j \), i.e., \( T_{L}^{k}(t_{i,k}) = \max(t_{i}(j) - t_{i,k}^*, 0) \).

\( N_{i,k} \) The number of commuters for each class, i.e., the average daily number of class \( k \) commuters from station \( O_i \).

\( \hat{n}_{i,j}^{k} \) Observed departure rate/boarding flow, i.e., the average daily number of class \( k \) commuters taking train \( j \) from station \( O_i \).

\( n_{i,j}^{0} \) Background passenger flow of train service \( j \) between \( O_i \) and \( O_{i+1} \), i.e., the average daily number of passengers taking train service \( j \) between \( O_i \) and \( O_{i+1} \) apart from the number of commuters \( \hat{n}_{i,j}^{k} \).

The results of commuter class identification are summarized in Table 2, where the train choice set is represented as \((j, j')\), indicating the range of available train services from the \( j \)th train to the \( j' \)th train. Figure 6 displays the observed departure rate \((\hat{n}_{i,j}^{k})\) of multi-class commuters from multiple origins. It is evident that commuters across all user classes do not uniformly choose the train service corresponding to their desired arrival time, primarily due to the cost associated with overcrowding. As detailed in Appendix D, the analysis of the equilibrium properties for a single origin and destination rail transit system reveals that the endogenous departure rate pattern signifies commuters’ trade-offs between schedule preference and in-vehicle crowding. During peak period, commuters’ departure rate initially increases and then decreases, reaching its peak at the time that aligns with their desired arrival time. Therefore, the observed user optimality in commuters’ departure time choices offers an opportunity to discern their preferences from the aggregated choice observations through equilibrium-based inverse optimization. In this context, the equilibrium model of departure time choice is essentially a system of simultaneous equations that account for the dynamic interactions among commuters’ travel choices.

As described in the last entry of Table 1, apart from the commuting demand related to the many-to-one rail corridor system, there is a significant number of morning rail passengers, averaging over 77,000 per day, departing from stations along the Batong Line to other destinations throughout the Beijing subway network, which has a nonnegligible impact on in-vehicle crowding cost. By matching these background passenger flows with train services based on their spatial-temporal information derived from smart card data and train timetable data, Figure 7 illustrates the average number of background passengers \((n_{i,j}^{0})\) for each train service between adjacent stations (Operation sections) along the Batong Line over 15 weekdays.

To measure commuters’ in-vehicle crowding perception preference towards the number of standees per square meter, the in-vehicle congestion degree \( f(n(t)) \) in Eq. (8) is determined by converting the number of passengers in the vehicle into the density of standing passengers using the following equation:

\[
f(n_{m,j}) = \max \left( \frac{n_{m,j} + n_{m,j}^0 - C_v}{C_a}, 0 \right),
\]  
(27)
where \( n_{m,j} = \sum_{l=1}^{m} \sum_{m-k} n_{l,j} \delta_{i,j} \), \( C_s \) and \( C_a \) denote the number of seats available and the standing area on each train, respectively.

Figure 6: Observed departure rate of multi-class commuters from multiple origins.

Figure 7: Background passenger flows along the Batong Line during the morning peak period.

Table 2: Summary of commuter classification.

<table>
<thead>
<tr>
<th>Origins</th>
<th>Class</th>
<th>( t_{i,k} )</th>
<th>( J_{i,k} )</th>
<th>( N_{i,k} )</th>
<th>Class</th>
<th>( t_{i,k}^* )</th>
<th>( J_{i,k} )</th>
<th>( N_{i,k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Class 1</td>
<td>8:35</td>
<td>(14, 31)</td>
<td>155</td>
<td>Class 4</td>
<td>9:35</td>
<td>(26, 46)</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>Class 2</td>
<td>9:15</td>
<td>(21, 46)</td>
<td>81</td>
<td>Class 5</td>
<td>8:15</td>
<td>(8, 31)</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>Class 3</td>
<td>8:55</td>
<td>(19, 41)</td>
<td>112</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Class 1</td>
<td>9:05</td>
<td>(24, 41)</td>
<td>60</td>
<td>Class 4</td>
<td>9:35</td>
<td>(26, 46)</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>Class 2</td>
<td>8:35</td>
<td>(14, 31)</td>
<td>134</td>
<td>Class 5</td>
<td>8:15</td>
<td>(5, 26)</td>
<td>109</td>
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<td>8:55</td>
<td>(16, 34)</td>
<td>100</td>
<td>Class 6</td>
<td>9:15</td>
<td>(24, 44)</td>
<td>56</td>
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<tr>
<td>3</td>
<td>Class 1</td>
<td>9:35</td>
<td>(29, 46)</td>
<td>57</td>
<td>Class 4</td>
<td>8:45</td>
<td>(16, 34)</td>
<td>87</td>
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<tr>
<td></td>
<td>Class 2</td>
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<td>Class 5</td>
<td>9:05</td>
<td>(19, 44)</td>
<td>95</td>
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</tbody>
</table>
During overcrowded morning peak period, commuters may experience passive boarding delays due to capacity constraints, or they may even actively choose subsequent train services to secure a seat (Chen et al., 2023; Mo et al., 2022; Zhu et al., 2021). In our dataset, the subway Batong Line operates with a train formation denoted as 6B, featuring a passenger capacity of 1440. The maximum load factor exceeded 130 percent during the peak period on the Beijing Subway Batong Line in 2017, as documented in the Beijing Transport Annual Report (BTAR, 2018), corresponding to over 1872 passengers per train. Since the chosen four origins are located upstream in the rail corridor system, as illustrated in Figure 7, the occupancy of all train services has not reached the maximum passenger capacity upon arriving at these four origin stations. For simplicity, the train services are assumed to arrive at these stations with a positive residual capacity, i.e., passengers will be able to board. However, capacity constraints significantly influence preference estimation when commuters’ choices are not solely driven by preference (Tian et al., 2007b), potentially resulting in self-selection bias. It is imperative to incorporate capacity constraints as well as passengers’ riding behavior (e.g., active boarding delay to secure a seat) into the empirical analysis of rail departure time choice. Moreover, capacity constraints will lead to increased waiting time at the platform, while active boarding delay may decrease the crowding cost. Future studies can gain a more comprehensive understanding of rail commuter behavior by incorporating these variations in the generalized travel cost.

5 Empirical Results

This section first demonstrates the estimation when measuring observed individual-level departure time choice data using econometric models of discrete choice, and then carries out the estimation through the inverse optimization method from an empirical study of the commuting corridor along the Beijing Subway Batong Line. The obtained estimation results are further compared with empirical evidence in previous studies. In the end, the functional forms of in-vehicle crowding level and variation in desired arrival time are discussed to identify their influence on estimation results.

5.1 Estimation from econometric models of discrete choice

For the econometric modeling of discrete choice, individuals’ generalized travel cost is defined as the same form of Eq. (4) but the in-vehicle crowding cost depends on the exogenous calculated passenger flow \( \hat{n}_{i,j} \), that is, the average daily number of passengers on the train service \( j \) between \( O_i \) and \( O_{i+1} \). The generalized travel cost experienced by commuter \( n \) belonging to class \( k \) taking train \( j \) from station \( O_i \) is given by:

\[
C_{n,i,j}^k = \alpha T(i,j) + \beta T_E^k(i,j) + \gamma T_L^k(i,j) + g \sum_{m=i}^{j} \hat{f}(\hat{n}_{m,j}^0) f_m, \quad \forall i \in \zeta, k \in K_i, j \in J_{i,k},
\]

(28)

where \( \hat{f}(\hat{n}_{m,j}^0) \) is the density of standees and can be formulated as:

\[
\hat{f}(\hat{n}_{m,j}^0) = \max \left(\frac{\hat{n}_{m,j}^0 - C_a}{C_a}, 0\right).
\]

(29)
Based on the observed individual-level commuting data of the many-to-one corridor system along the Beijing Subway Batong Line, the Biogeme, an open-source software package for econometric models of discrete choice developed by EPFL (Bierlaire, 2018), is employed to estimate the preference parameters from the following multinomial logit model:

$$p_{n,i,j}^k = \frac{\exp(-C_{n,i,j}^k)}{\sum_{s \in J_{i,k}} \exp(-C_{n,i,s}^k)}, \forall i \in \zeta, k \in K, j \in J_{i,k}. \quad (30)$$

Considering potential correlation in unobserved factors and random preference variations across individuals, we further use the mixed logit to estimate the generalized cost function, where the preference parameters are assumed to be normally distributed, and the number of Halton draws has been set to 1000 in the maximum simulated likelihood estimation.

Table 3: Estimation results using multinomial logit model and mixed logit model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>t-test</th>
<th>Mean</th>
<th>t-test</th>
<th>St.dev</th>
<th>t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.000357*</td>
<td>1.92</td>
<td>0.000435*</td>
<td>1.78</td>
<td>0.007401***</td>
<td>15.48</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.001400***</td>
<td>104.99</td>
<td>0.001524***</td>
<td>83.15</td>
<td>0.000004</td>
<td>0.08</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.001823***</td>
<td>102.75</td>
<td>0.001917***</td>
<td>97.77</td>
<td>0.000024</td>
<td>0.36</td>
</tr>
<tr>
<td>$g$</td>
<td>-0.000011*</td>
<td>-1.81</td>
<td>-0.000009*</td>
<td>-1.65</td>
<td>0.000079**</td>
<td>1.99</td>
</tr>
<tr>
<td>Init log-likelihood</td>
<td>-88212.12</td>
<td></td>
<td>-88212.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final log-likelihood</td>
<td>-70653.04</td>
<td></td>
<td>-70083.65</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rho square</td>
<td>0.20</td>
<td></td>
<td>0.21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rho bar square</td>
<td>0.20</td>
<td></td>
<td>0.21</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

***denotes significance level at 1%, ** at 5% and * at 10%.

Table 3 summarizes the estimation results using multinomial logit model and mixed logit model, respectively. Both models yield a negative $g$, which implies that crowding congestion leads to a positive utility and commuters are more willing to travel when the train is more crowded. This estimation bias primarily stems from the violation of the fundamental exogeneity assumption concerning the explanatory variables in econometric models of discrete choice. The aggregated measurements of in-vehicle crowding degree, as detailed in Table 1, reflect the choice outcomes of smart card holders, thus introducing simultaneity effects when directly using these endogenous statistics as explanatory variables. Typically, endogeneity in discrete choice models can be addressed through the judicious utilization of proxy variables, instrument variables, latent variables, and multiple indicators, employing either simultaneous or two-stage estimation procedures (Guevara and Ben-Akiva, 2010; Guevara, 2015). Based on the observations drawn exclusively from smart card data, we note that during morning peak period, as shown in Figures 6 and 7, in-vehicle passenger flows generally increase as the desired arrival time approaches. This correlation suggests the potential for using time of schedule delays as instruments to address the endogeneity of in-vehicle crowding costs. Following the two-stage estimation method proposed by de Grange et al. (2024), we attempt to address such endogeneity in crowding cost measurement with instrument variables of schedule delays, and preliminarily resolve the sign bias in crowding perception estimates (see Appendix E). However, schedule delay instruments can only be applied to homogenous commuters or specific commuter classes, within whose range of arrival time choices the in-vehicle crowding cost are highly correlated with the time of schedule delay. This
limitation makes it difficult to account for travel time preference heterogeneity, which is crucial for a more precise estimation and understanding. Therefore, given the limited behavioral information and lack of robust exogenous identifiers when using smart card data, constructing compelling substitutes for endogenous variables is rather challenging, and inappropriate treatments of endogeneity may further exacerbate estimation problems (Guhl, 2019).

Given the limited information available in the smart card data for addressing endogeneity, we shift our focus to examining the dynamics within the panel structure of individuals’ travel behavior data. Capturing the dynamic aspects of behavior involves specifying that representative utility in each period is contingent on observed variables from previous periods, offering a non-statistical remedy to alleviate concerns related to simultaneity effects (Train, 2009; Zaefarian et al., 2017). In the day-to-day choices of departure times within a period of $D$, commuters may adjust their train services based on previous days’ travel experience to avoid longer travel time and more severe in-vehicle overcrowding. We assume that commuters’ departure time choices on a given day are primarily influenced by their travel experiences from the day before, that is, commuters may evaluate their travel cost based on the measurements of various preference ingredients/components from the previous day. As the travel time and in-vehicle crowding degree of all train alternatives from the previous day are dependent on commuters’ departure time choices for that day, individuals’ travel behavior on a given day can be further linked with their past choices. Therefore, we use the one-day lagged measurements of preference ingredients as references for commuters’ choices and the generalized travel cost on day $t$ is defined as:

$$
C_{n,i,j,t}^k = \alpha T_{t-1}^k (i, j) + \beta \tilde{T}_{E,t-1}^k (i, j) + \gamma \tilde{T}_{L,t-1}^k (i, j) + g \sum_{m=i}^f \tilde{\tau}_m,
$$

where the subscript $t-1$ represents the lagged variables of corresponding cost measurements from the previous day.

Trip data from smart card holders with a complete commuting record over 15 weekdays are sampled, constructing a balanced panel dataset that includes 6,810 trip records from 454 commuters. Subsequently, individuals’ commuting patterns are clustered to identify their habitual temporal travel patterns and discern their underlying travel time preferences, as detailed in Section 4.1 (The classification results are summarized in Appendix F.1). Following this, we estimate the parameters in Eq. (31) using the multinomial logit model. According to the estimation results presented in Table 4, all signs of the estimated parameters are consistent with the underlying rational behavioral assumption. Although the parameter of $\alpha$ exhibits low significance, considering the dynamics in individuals’ departure time choice behavior can help mitigate the estimation bias. Furthermore, we also consider the existence of inertia or habit formation in individuals’ day-to-day decision-making, where travelers are inclined to persist with the previously chosen alternative unless another alternative offers substantially higher utility to justify a switch (Train, 2009). This behavioral inertia is represented as an explanatory variable that indicates the proportion of times the train alternative has been chosen previously (refer to the details in Appendix F.2). The estimated parameters for schedule preference and crowding perception also exhibit appropriate signs and the negative value of parameter associated with behavioral inertia suggests that commuters tend to stick with the train services they had chosen before. However, as commuters’ choices are not observed from their initial decision-making situations, such models remain susceptible to endogeneity, known as the initial condition problem in dynamic choice
models. Addressing this issue requires further integrating potential approaches, as demonstrated by Heckman and Singer (1986).

Table 4: Estimation results from panel data analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Multinomial logit</th>
<th>Mixed logit (Normal distr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>t-test</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.000354</td>
<td>0.60</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.002325***</td>
<td>59.21</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.002251***</td>
<td>56.46</td>
</tr>
<tr>
<td>$g$</td>
<td>0.000029**</td>
<td>2.26</td>
</tr>
<tr>
<td>Init log-likelihood</td>
<td>-19224.69</td>
<td></td>
</tr>
<tr>
<td>Final log-likelihood</td>
<td>-14637.99</td>
<td></td>
</tr>
<tr>
<td>Rho square</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>Rho bar square</td>
<td>0.24</td>
<td></td>
</tr>
</tbody>
</table>

As examined by Small (1982), the assumption of independence from irrelevant alternatives (IIA) in the logit formula appears suspicious due to the natural sequential ordering of departure alternatives. In this context, unobserved random factors may exhibit correlation among nearby train alternatives. If IIA holds in the choice setting, the parameter estimates derived from a subset of alternatives should be approximately equal to those obtained using the complete set of alternatives. We re-estimate Eq. (31) on restricted choice subsets, successively removing one train alternative, and then calculate the Hausman-McFadden statistic\(^3\) to test the assumption of IIA for each of the alternatives. However, out of the 46 tests conducted, merely 18 support the null hypothesis of IIA at a 10% significance level. It is noteworthy that violations intensively occur when removing train alternatives during periods of overcrowding. This suggests a higher likelihood of interdependence among unobserved random factors across alternatives, particularly in proximity to the desired arrival time. This test outcome is superior to the results obtained when assuming that commuters originating from the same station share common travel time preferences (refer to Appendix F.3), indicating that considering heterogeneity in travel time preferences can help accommodate interdependence over train alternatives. Furthermore, given our assumption of dynamics in the observed cost measurements in Eq. (31), it is plausible that dynamics also exist in the unobserved factors. Therefore, we also use the mixed logit that allows the random terms to be correlated over different train alternatives. As shown in Table 4, assuming a normal distribution for the preference parameters, the mean of parameter estimates suggests appropriate signs, while the non-zero standard deviations of the mixing distribution also imply a violation of IIA (Train, 2009).

Through the estimation results obtained from discrete choice models using panel data, it is evident that individual commuters tend to strategically reduce their travel disutility in subsequential choices. This behavioral tendency aligns with the assumption of user optimality in the equilibrium model, thereby strengthening the validity of inferring commuters’ preferences from the aggregate observations.

---

\(^3\) Hausman and McFadden (1984) provided a specification test for the IIA assumption of the multinomial logit model by constructing the following asymptotically distributed chi-square statistic:

$$
(\hat{\omega}_s - \hat{\omega}_o)(\text{cov}(\hat{\omega}_s) - \text{cov}(\hat{\omega}_o))^T(\hat{\omega}_s - \hat{\omega}_o)^T \rightarrow \chi^2(m),
$$

where $\hat{\omega}_s$ and $\hat{\omega}_o$ are the parameter estimates derived from the subset of alternatives and the overall set of alternatives, respectively, with $\text{cov}(\cdot)$ representing the covariance matrix of the estimates. The degree of freedom is equal to the dimension of $\hat{\omega}_o$. 

\[\]
However, the data-driven nature of discrete choice models makes them highly dependent on the quality of individual-level data, and the inherent sampling bias in estimating individual-level observations may be further magnified over aggregations, resulting in inaccurate estimation (Guarda and Qian, 2023). To address the limitations on individual-level observations solely relying on smart card data, the proposed equilibrium-based inverse optimization method resorts to measuring system-level aggregations, and thus becomes less susceptible to the sampling bias.

5.2 Estimation through inverse optimization method

To reveal the heterogeneity in commuters’ travel time preferences, we first consider the urban rail commuting corridor system with homogenous commuters and demonstrate the performance of the SAB solution method for inverse optimization. Then, we extend our analysis to incorporate the heterogeneity of commuters’ travel time preferences. Specifically, we examine the urban rail commuting corridor with multiple classes of commuters.

5.2.1 Rail commuting corridor with homogenous commuters

The urban rail commuting corridor system with four origins of Tu Qiao, Liyuan, Guoyuan and Tongzhou Beiyuan, shown in Figure 2, is used to demonstrate the estimation of schedule preference and crowding perception. It is assumed that commuters originating from the same station share common travel time preferences. Based on the arrival time feature depicted in Figure 8, when treating the commuters from each station as a single class, we set the arrival time that aligns with the peak value as their desired arrival time and the train services available during the time range of arrival time choices as their train choice set. To be more specific, the desired arrival times for commuters from Tu Qiao, Liyuan, Guoyuan and Tongzhou Beiyuan are 8:35, 8:45, 8:35 and 8:45, respectively. The corresponding train choice sets are (8,46), (5,46), (5,46) and (3,46). The total number of commuters originating from each station is set as the summation of the counts across all classes as listed in Table 2. In addition, each train service of Batong Line offers approximately 256 seats ($C_s$) and provides an area of approximately 275 square meters for standing ($C_a$). For the parameters related to solution algorithms, we set the parameters $\delta = 0.0002$, $\delta^* = 0.0001$ and $\eta = 0.0001$, respectively, and the initial preference parameter value of $\omega_0 = (\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}, \tilde{\delta})$ is set as $\omega_0 = (0.01, 0.01, 0.01, 0.01)$.

![Figure 8: Arrival time features of homogeneous commuters from multiple origins.](image-url)
Figures 9(a) and 9(b) illustrate the iterative process of the proposed SAB solution method by showcasing the changes in the upper-level objective value and upper-level preference parameters \( \tilde{e}_p \). It takes 70 iterations to converge to the solution of \( \tilde{\alpha}^* = 0.000267, \tilde{\beta}^* = 0.000602, \tilde{\gamma}^* = 0.000479 \) and \( \tilde{g}^* = 0.000025 \), and the optimal upper-level objective value is 4326.06. Figure 9(a) also compares the upper-level objective value of computed equilibrium and approximation from sensitivity analysis at each iteration. It is worth noting that the sensitivity analysis of the rail corridor commuting equilibrium model can approximate the equilibrium state with high accuracy. Furthermore, Figure 10 depicts the equilibrium departure rates with respect to \( \tilde{\omega}^* \), along with the observed departure rates of train services for each station during the morning peak period.

Figure 9: Iterative process: (a) upper-level objective value and (b) preference parameters.

Figure 10: Equilibrium departure rates along with observed departure rates.
We also set different values of $\bar{\omega}_0$ to investigate the impact of the initial preference parameter value on the solution of inverse optimization. Figures 11(a) to 11(d) show the iterations of the upper-level objective value and upper-level preference parameters given $\bar{\omega}_0 = (0.2, 0.2, 0.2, 0.2)$ and $\bar{\omega}_0 = (0.4, 0.3, 0.2, 0.1)$, respectively. It can be observed that various initial parameter values can converge to the same solution representing an identical commuting equilibrium state, and larger initial preference parameter values may lead to a slower convergence due to a predetermined neighborhood radius $\delta$ at $\bar{\omega}_p$ of each iteration. As the signs of $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ in $\bar{\omega}$ do not influence the convexity of Eq. (13), we proceed to relax the non-negative constraints on these three parameters and reset the initial parameter values as $\bar{\omega}_0 = (-0.01, -0.01, -0.01, 0.01)$. Figures 11(e) and 11(f) illustrate that the optimization process can also converge to the same estimated preference parameter values.

Figure 11: Convergence performance given different $\bar{\omega}_0$.

To validate the effectiveness of the inverse optimization-based parameter estimation, we conduct experiments using synthetic data generated under an ideal setting, where the departure rate counts align perfectly with the logit-based SUE. Based on the same commuting corridor scenario as mentioned above, the synthetic departure rate distribution is generated under a set of ground truth preference parameters $\bar{\omega}^s = (0.000300, 0.000600, 0.000500, 0.000030)$. Figures 12(a) and 12(b) show the iterative process of parameter estimation using synthetic data. The results demonstrate that the inverse optimization solution can accurately converge to the actual preference parameters. Furthermore, to graphically demonstrate the properties of the upper-level objective function near the actual parameter values, Figures 12(c) and 12(d) depict the contour lines of $\bar{F}(\bar{n}^*(\bar{\omega}))$ with respect to two of the preference parameters, while fixing the values of the other preference parameters at their actual values. The ground truth values of the preference parameters are indicated by dashed lines in each figure. It can be seen that near the optimum point, $\bar{F}(\bar{n}^*(\bar{\omega}))$ changes smoothly and is convex with respect to $\bar{\alpha}$ and $\bar{g}$. However, the convexity of the upper-level objective value cannot be always guaranteed, as it is evident that the upper-level objective function is non-convex with respect to changes in parameters $\bar{\beta}$.
and $\tilde{\gamma}$. Despite the inherent non-convexity in the equilibrium-based inverse optimization, the solution obtained using the SAB solution method can converge to the predetermined optimum.

Figure 12: Iterative process based on synthetic data and non-convexity of the upper-level objective function $\tilde{F}(\mathbf{n}(\mathbf{w}))$.

To investigate the impact of randomness in departure rate distribution on preference parameters estimates, we introduce random errors to the departure rate counts of ground truth preferences through a series of Monte Carlo experiments. Assuming a normally distributed errors, random errors are set with standard deviations equal to 3%, 5%, and 10% of the values of the departure rate counts, respectively. For each level of random error distribution, 20 replicates are generated for experiments to estimate the preference parameters and compute the estimation bias. Figures 13(a), 13(b) and 13(c) illustrate the parameter estimation bias under three different random error distributions, and Figure 13(d) displays the corresponding distribution of upper-level objective function values. The experimental results indicate that the parameter estimation results for $\tilde{\gamma}$, $\tilde{\beta}$ and $\tilde{g}$ are roughly unbiased across different random error distributions. However, the estimates for $\tilde{\alpha}$ exhibit some variability, although the extent of the bias is relatively small compared to the actual value of the parameter. This may stem from insufficient consideration of the impact of train capacity constraints and commuters’ riding behavior on travel time, as discussed in Section 4.2, which also results in endogeneity. As the proposed commuting equilibrium model primarily addresses the simultaneity effects between commuters’ departure time choices and in-vehicle crowding, it may not comprehensively capture the complexity and noise inherent in real-world behavioral data. Therefore, it is challenging to guarantee the unbiasedness of the estimates due to idealized behavioral assumption and limited decision information. Future research could enhance the unbiasedness of estimation by incorporating more robust
optimization techniques (e.g., Aswani et al., 2018), sophisticated behavioral modeling, and integration with multi-source data.

Figure 13: Bias in preference parameter estimation and corresponding upper-level objective values under Monte-Carlo experiments.

5.2.2 Rail commuting corridor with multi-class commuters

We further consider the rail commuting corridor system with multi-class commuters who exhibit heterogeneity in their travel time preferences. As outlined in Table 2, commuters originating from each station are clustered into multiple distinct classes with respective desired arrival times and train choice sets. Under the same parameters settings as stated in Section 5.2.1 for the solution algorithms, Figure 14 shows the convergence of the upper-level objective value and upper-level preference parameters in the iterative process which takes 80 iterations to converge to the solution of $\tilde{\alpha}^*=0.001144$, $\tilde{\beta}^*=0.001890$, $\tilde{\gamma}^*=0.002123$ and $\tilde{g}^*=0.000103$ with the optimal upper-level objective value of 971.80.

Compared to homogenous commuters, the estimates for multi-class commuters are larger, indicating a lower randomness in choices and a better understanding of time-of-day commuting costs. It is also evident that the schedule delay costs for multi-class commuters, i.e. the value of $\tilde{\beta}^*$ and $\tilde{\gamma}^*$, suggest the penalty for arriving late is slightly higher than that for arriving early, which contrasts with that for homogenous commuters. This discrepancy is attributed to the larger train choice set available for homogenous commuters, allowing them greater flexibility in departure time choices. As a result, the estimation for homogenous commuters exhibits a lower sensitivity to schedule delay costs. Furthermore, the specific desired arrival times for different commuter classes exert a strong influence on their preferences regarding early or late arrival, thereby significantly impacting the estimation results. Moreover, the optimal upper-level objective value for multi-class commuters is considerably smaller than that of homogenous commuters, indicating a superior fit of the multi-class commuter equilibrium with the observed commuting data. Figure 15 illustrates the equilibrium departure rates under the estimated preference parameters obtained from inverse optimization and discrete choice model of multinomial logit using panel dataset, respectively. Since the panel commuting data are sourced from individuals with a complete commuting record spanning 15 weekdays, commuters with specific rail trip frequencies may exhibit different time utility preferences, thus contributing to variations in preference estimation. Although there is a slightly larger deviation from the observed departure rate distribution, measuring the day-to-day dynamics in individual travel behavior might help uncover an equilibrium departure rate pattern similar to that from inverse optimization.
5.3 Empirical comparisons with previous studies

In transit crowding valuation studies, a common objective is to estimate a crowding multiplier, which represents the ratio between travel time parameters under crowded and uncrowded in-vehicle conditions. The typical approach involves defining a crowding attribute that interacts with travel time to capture the impact of heightened crowding discomfort during transit trips (Tirachini et al., 2016). In this paper, the crowding multiplier $CM$ for rail commuters is defined as the ratio between the marginal time utility when standing and the marginal time utility under uncrowded conditions with no standees:

$$ CM = \frac{\tilde{\alpha}^* + \tilde{\gamma}^* f(n)}{\tilde{\alpha}^*},$$  \hspace{1cm} (32)

where $f(n)$ is the density of standees per square meter.

During the morning peak period of Batong Line, the most crowded train services, specifically the 8th to the 32nd train services, have an average in-vehicle passenger density ranging from 3 to 6 standees per square meter. Referring to the estimated preference parameters from the rail commuting corridor with multi-class commuters along the Batong Line, the crowding multiplier is calculated to be
from 1.27 to 1.54. This suggests that during crowded conditions, the travel time disutility for commuters along the Beijing Subway Batong Line is evaluated as approximately 27-54% higher than during uncrowded conditions.

Although the value of $CM$ depends to largely on local circumstances and modeling methods with either stated preference or revealed preference survey data, static or dynamic models, our focus is on comparing the international preferences regarding the perception of crowding when using public transport. According to the meta-analysis of 208 time-based rail crowding valuations compiled from 15 British stated preference studies (Wardman and Whelan, 2011), the time multiplier averages 1.19 for sitting while averages 2.32 for standing, which implies an integrated value of $CM$ around 1.95. The stated preference studies conducted in Seoul indicate a crowding multiplier of up to 1.50 with 5 standees per square meter (Shin et al., 2021), which aligns with our findings in Beijing. In the Santiago metro system, the crowding multiplier ranges from 1.10 to 1.15, with the number of standees varying from 0 to 6 per square meter (Tirachini et al., 2017), similar to the crowding multiplier reported for Paris from all public transport modes (Kroes et al., 2014). From the estimation based on the revealed preference approach, the estimated crowding multiplier in Hong Kong is 1.265, with an additional passenger per square meter contributing an average increase of 0.119 to the multiplier (Hörcher et al., 2017). Furthermore, during the morning commute in Singapore, the crowding multiplier can reach as high as 1.55 with a density of 3 standees per square meter (Tirachini et al., 2016).

From the perspective of dynamic travel modeling in massive transit systems, though there is limited literature specifically evaluating in-vehicle crowding cost, most of the related studies are based on the classic scheduling model proposed by Small (1982) and Vickrey (1969). This common framework facilitates the comparison of empirical findings on schedule preferences across various studies. In our study, the schedule delay penalty ratio of arriving late over arriving early (i.e. $\gamma^*/\beta^*$) is about 1.12 for rail commuters along Batong Line, indicating a slightly higher penalty of arriving late over arriving early. This result is close to the previous findings of 1.08 derived from revealed mobility data on Beijing Subway Line 13 (Tian et al., 2009) while exhibiting significant differences from the estimates obtained through stated preference research conducted on the Beijing subway system, specifically 1.84 in Yan et al. (2016) and 1.46 in Li et al. (2018). Furthermore, the schedule delay penalty ratio varies between 1.32 and 2.37 for heterogenous rail passengers in Nanjing (Cheng et al., 2020), and 1.30 and 1.04 respectively for revealed preference and stated preference from multimodal morning commuters in Santiago (Lizana et al., 2021).

### 5.4 Impact of functional form of crowding on estimation

The functional form of $f(n(t))$, representing the in-vehicle crowding degree in relation to passenger density, can directly influence the estimation of commuters’ in-vehicle crowding perception since the marginal in-vehicle crowding disutility is determined by the product of $g$ and $f(n(t))$. In the aforementioned modeling and estimation, as depicted in Eq. (27), the functional form of $f(n_{m,j})$ is linearly dependent on the density of standees. In order to examine the impact of different functional forms for this relationship, we further consider the square and square root of $f(n_{m,j})$ with respect to the density of standees, respectively:

$$f(n_{m,j}) = \max \left( \frac{(n_{m,j} + n^0_{m,j} - C_x)}{C_a}, 0 \right), \quad (33a)$$
\[ f(n_{m,j}) = \max \left\{ \sqrt{\frac{n_{m,j} + n_{m,j}^0 - C_i}{C_a}}, 0 \right\}. \] (33b)

The estimation results obtained by employing the quadratic and square root functional forms of \( f(n_{m,j}) \) from the rail commuting corridor on the Batong Line, which accommodates multi-class commuters as demonstrated in Section 5.2.2, are presented in Table 5. According to the schedule delay penalty ratio of \( \hat{\gamma}^*/\hat{\beta}^* \), it can be seen that schedule preference exhibits slight change compared to that estimated under linear formulated \( f(n_{m,j}) \). However, the magnitude of parameter \( \hat{g}^* \), which represents the preference for crowding perception, varies evidently due to the exponent of the crowding function. Specifically, in contrast to the linear formulated \( f(n_{m,j}) \), the quadratic crowding results in a smaller value of crowding perception, while the square root crowding yields relatively larger values. Meanwhile, the square root congestion degree results in a higher value of \( CM \), whereas the quadratic congestion degree leads to a lower-level of crowding discomfort when there are 6 standees per square meter. Furthermore, both the linear and quadratic crowding functions exhibit a slightly better fit over the square root one according to the upper-level objective value, but the linear relationship tends to converge faster with fewer iterations. Therefore, the linearity assumption may be fairly plausible for modeling the in-vehicle crowding. Many empirical results also pointed towards a linear relationship between in-vehicle crowding costs and passenger density (Prud’homme et al., 2012; Haywood and Koning, 2015; Tirachini et al., 2016), but did not find enough empirical support for complex specifications by examining various crowding cost functions (Whelan and Crockett, 2009; Hensher et al., 2011).

Table 5: Estimation results under different functional forms of crowding.

<table>
<thead>
<tr>
<th>Functional form</th>
<th>( \hat{\alpha}^* )</th>
<th>( \hat{\beta}^* )</th>
<th>( \hat{\gamma}^* )</th>
<th>( \hat{g}^* )</th>
<th>( CM )</th>
<th>( \hat{\gamma}^<em>/\hat{\beta}^</em> )</th>
<th>Upper-level objective value</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.001144</td>
<td>0.001890</td>
<td>0.002123</td>
<td>0.000103</td>
<td>1.54</td>
<td>1.12</td>
<td>971.80</td>
<td>80</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.001044</td>
<td>0.001878</td>
<td>0.002061</td>
<td>0.000009</td>
<td>1.31</td>
<td>1.10</td>
<td>969.69</td>
<td>204</td>
</tr>
<tr>
<td>Square root</td>
<td>0.001321</td>
<td>0.001882</td>
<td>0.002125</td>
<td>0.000385</td>
<td>1.71</td>
<td>1.13</td>
<td>1008.11</td>
<td>91</td>
</tr>
</tbody>
</table>

5.5 Impact of variation in desired arrival time on estimation

The determination of desired arrival time holds significant sway over the overall estimation of commuters’ travel preferences. As stated in Section 4.1, the desired arrival time for each commuter class is determined based on the peak value of their arrival time feature. Nevertheless, it is important to note that for certain arrival time features, there may not be a distinct peak value, and a higher frequency of arrival times does not necessarily indicate the most preferred arrival time for each commuter class. Consequently, there exists significant uncertainty in determining commuters’ desired arrival time. To explore the impact of subtle changes in desired arrival time on the estimation of various travel preference attributes, we attempt to set different quantiles of multiple commuter classes’ arrival time features as their respective desired arrival times, which allows us to investigate the specific influence of these variations on the estimation results.

The arrival time feature vector of class \( k \) commuters from origin \( i \) is denoted as \( F_{i,k} \), and each vector contains 18 entries, one for each \( f_{i,k}(\text{hour}(t), \text{minute}(t)) \) representing the class-specific arrival time frequency for every 10 minutes from 7:00 am to 10:00 am over 15 weekdays.
\[
F_{i,k} = \left[ f_{i,k}(7,10), f_{i,k}(7,20), \ldots, f_{i,k}(9,60) \right], \quad \forall i \in \zeta, k \in K_i, \tag{34}
\]

To depict the arrival time frequency distribution of each \(F_{i,k}\), we define the following equation, denoted as \(F(x_{i,k}^{\text{hour}}, x_{i,k}^{\text{minute}})\), as the sample distribution function of \(F_{i,k}\):

\[
F(x_{i,k}^{\text{hour}}, x_{i,k}^{\text{minute}}) = \frac{\sum_{\text{hour}(t)=7}^{\text{hour}(t)=10} f_{i,k}(\text{hour}(t), \text{minute}(t))}{\sum_{\text{hour}(t)=7}^{\text{hour}(t)=10} f_{i,k}(\text{hour}(t), \text{minute}(t))}, \quad \forall i \in \zeta, k \in K_i. \tag{35}
\]

Obviously, \(F(x_{i,k}^{\text{hour}}, x_{i,k}^{\text{minute}})\) represents the cumulative frequency from the initial 10 minutes of 7 AM up to the time of \(\text{hour}(t) = x_{i,k}^{\text{hour}}\) and \(\text{minute}(t) = x_{i,k}^{\text{minute}}\). \(t_{i,k}^{p*}\) is further defined as the \(p\) quantile of \(F_{i,k}\), that is, \(t_{i,k}^{p*}\) is the smallest among all \(x_{i,k}^{\text{hour}}\) and \(x_{i,k}^{\text{minute}}\) for which \(F(x_{i,k}^{\text{hour}}, x_{i,k}^{\text{minute}})\) is greater than or equal to \(p\):

\[
t_{i,k}^{p*} = \inf \left\{ (x_{i,k}^{\text{hour}}, x_{i,k}^{\text{minute}}) : F(x_{i,k}^{\text{hour}}, x_{i,k}^{\text{minute}}) \geq p \right\}, \quad \forall i \in \zeta, k \in K_i. \tag{36}
\]

Table 6: Estimation results under different desired arrival times.

<table>
<thead>
<tr>
<th>(p) quantile</th>
<th>(\tilde{\alpha}^*)</th>
<th>(\tilde{\beta}^*)</th>
<th>(\tilde{\gamma}^*)</th>
<th>(\tilde{g}^*)</th>
<th>CM</th>
<th>(\tilde{\gamma}^<em>/\tilde{\beta}^</em>)</th>
<th>Upper-level objective value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.001073</td>
<td>0.001925</td>
<td>0.001865</td>
<td>0.000072</td>
<td>1.40</td>
<td>0.97</td>
<td>1498.63</td>
</tr>
<tr>
<td>0.55</td>
<td>0.001091</td>
<td>0.001688</td>
<td>0.001943</td>
<td>0.000074</td>
<td>1.41</td>
<td>1.15</td>
<td>2142.69</td>
</tr>
<tr>
<td>0.6</td>
<td>0.001108</td>
<td>0.001681</td>
<td>0.002091</td>
<td>0.000085</td>
<td>1.46</td>
<td>1.24</td>
<td>1604.25</td>
</tr>
<tr>
<td>0.65</td>
<td>0.001125</td>
<td>0.001636</td>
<td>0.002152</td>
<td>0.000093</td>
<td>1.50</td>
<td>1.32</td>
<td>1531.08</td>
</tr>
<tr>
<td>0.7</td>
<td>0.000783</td>
<td>0.001168</td>
<td>0.002200</td>
<td>0.000058</td>
<td>1.44</td>
<td>1.88</td>
<td>3776.25</td>
</tr>
<tr>
<td>Peak value</td>
<td>0.001144</td>
<td>0.001890</td>
<td>0.002123</td>
<td>0.000103</td>
<td>1.54</td>
<td>1.12</td>
<td>971.80</td>
</tr>
</tbody>
</table>

Table 6 presents the estimated results using various values of \(t_{i,k}^{p*}\) as the desired arrival time, respectively. It can be observed that the schedule delay costs exhibit a gradual change as the increase of \(t_{i,k}^{p*}\). Notably, there is a decrease in the marginal utility cost of schedule delay early and an increase in the marginal utility cost of schedule delay late, which suggests an increasing schedule delay penalty ratio of \(\tilde{\gamma}^* / \tilde{\beta}^*\). Furthermore, the trip time cost and the in-vehicle crowding cost demonstrate a relatively consistent change with the variation of desired arrival time, since the duration of rail commuting is typically influenced by the level of crowdedness experienced during the journey. As the \(p\) quantile increases, the crowding multiplier increases first and then decreases when considering 6 standees per square meter in-vehicle, and a relatively higher level of crowding discomfort is evaluated when the desired arrival time is set to the peak value of the arrival time feature. Moreover, although arrival times with the highest trip frequency may not fully capture individual arrival time preference, the peak value tends to offer a better fit compared to the other quantiles, as indicated by the upper-level objective values. These findings highlight the significant impact of determining commuters’ travel time preferences on estimating all attributes, and it would be relevant to examine commuters’ travel time
preferences from more substantial evidence, such as integrating with state preference experiments or multisource data, which also presents a potential issue for future research.

6 Conclusion

The growing availability of massive transit mobility data provides opportunities to better measure or evaluate users’ travel preferences in real-world situations. While discrete choice models are seen as a promising approach for understanding individual tastes in travel behavior studies, addressing endogeneity in system-level preference measurements poses a challenge due to the constraints of limited exogenous information coupled with smart card-based travel records. Additionally, the sampling bias of individual choices from large-scale mobility dataset may introduce inaccurate or even misleading estimation as well. To address these limitations, we employ an equilibrium-based inverse optimization method to uncover the cost/disutility endogeneity inherent in commuters’ decision-making, particularly when weighing trade-offs among various preference considerations, through the measurement of aggregated cost measurements. By assuming user optimality in observed choice behavior, we aim to optimize a set of preference parameters in the commuting equilibrium model so that the resulting equilibrium state approximates the user optimality observed in real-world data. To achieve this, we have formulated a bi-level programming problem that draws upon the legitimacy of the inverse optimization modeling method, in which the lower-level is the rail corridor commuting equilibrium model and the upper-level is the parameter optimization problem. Furthermore, to capture the heterogeneity and randomness in actual commuting observations, the commuting equilibrium model primarily incorporates multi-class commuters with respective desired arrival times and choice set of train services, and the corresponding equilibrium condition satisfies the multinomial logit-based departure time choice principle, which allows a more comprehensive representation of commuters’ endogenous departure time decision-making. Based on the sensitivity analysis of the lower-level rail corridor commuting equilibrium model, a sensitivity analysis-based line search solution method has been proposed to solve the bi-level programming model of the inverse optimization.

The smart card data and train timetable data from the rail corridor along the Beijing Subway Batong Line in October 2017 are synthesized as a case study to estimate rail commuters’ departure time choice preferences during morning peak periods. The rail commuters from the four main origins of Tu Qiao, Liyuan, Guoyuan and Tongzhou Beiyuan to Sihui Dong station are clustered into multiple classes with different travel time preferences according to their habitual temporal travel patterns, and then the rail transit data from multi-class commuters are aggregated to estimate their schedule preference and crowding perception through inverse optimization. Based on the estimation results of the rail commuting corridor with multi-class commuters, the crowding multiplier is calculated to be in the range of 1.27 to 1.54 when the average in-vehicle passenger density varies from 3 to 6 standees per square meter, indicating that the disutility of unit travel time for commuters along the Beijing Subway Batong Line is evaluated approximately 27 to 54 percent higher during morning peak period. The schedule delay penalty ratio of arriving late over arriving early is about 1.12 for rail commuters along the Batong Line, indicating a slightly higher penalty of arriving late over arriving early. These values are also compared with previous studies from different cities and countries. To assess the impact of the measurement of in-vehicle crowding degree on crowding perception, the quadratic and square root functional forms of crowding are further examined and compared with the linear one. However, our findings indicate that the simple linear specification is indeed competent for modeling the in-vehicle crowding degree.
Several important issues warrant in-depth exploration in the future. Firstly, since commuters’ time-of-use decisions are primarily derived from smart card-based observations, it is crucial to investigate commuters’ behavioral preferences with more comprehensive evidence. This entails further consideration of their entire travel process, including activities at both ends of the rail commute, e.g., using synthesized data. The incorporation of more robust optimization techniques to address noisy measurements (Aswani et al., 2018), along with more reliable exogenous identifiers such as instrumental variables into commuters’ generalized cost function, may further enhance the accuracy of user preference estimates. Secondly, it is important to note that our analysis is limited to considering heterogeneity in travel time preferences, without incorporating heterogeneity in commuters’ taste parameters as explored in prior research (e.g., Haywood and Koning, 2015; Hong et al., 2017; Haywood et al., 2017; Tian et al., 2021). The exclusion of heterogeneous variables increases the likelihood of violating the IIA assumption in logit-based SUE. Thirdly, boarding constraints indeed have a substantial impact on the preference estimation if commuters’ choices are not solely driven by their preferences. It is imperative to incorporate capacity constraints as well as passengers’ riding behavior into the empirical analysis of rail departure time choice for further research. Finally, while this paper focuses on commuters’ departure time choice behavior along a many-to-one rail transit corridor, the proposed methodology can be applied to many-to-many corridor scenarios, and provide a basis for extensions to more general cases such as integrating departure time choices with route choices for a more comprehensive rail commuting preference estimation.

Model Implementation and Data Availability

The Python code and necessary data for implementing the methodology discussed in this paper can be found at https://github.com/XuPu0126/InverseOptimizationBasedPreferenceEstimation to facilitate result replication. The raw smart card data and train timetable data from the Beijing Subway Batong Line are protected and unavailable due to data privacy laws.

Acknowledgement

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Appendix A. Proof of The Convexity of Eq. (10a)

The first term of the objective function (10a) is strictly convex over $n_{i,j}^{k}$ since its Hessian matrix is positive definite. If the remaining two terms in the objective function are also convex, the summation of them is a strictly convex function. Since the third term of the objective function (10a) is the summation of linear functions, it suffices to investigate whether the second term satisfies the convexity conditions.

Let $G_m$ denote one of the terms in the second summation of Eq. (10a):

$$G_m = \left( \sum_{j \in J} g \sum_{\rho_{i,j}^{k}} \int_{0}^{\xi} \sum_{\rho_{l,k}^{l}} \sum_{\rho_{i,j}^{l}} f(\nu) d\nu \right) \tau_m, \quad \forall m \in \zeta.$$  \hspace{1cm} (A.1)
The gradient of $G_m$ with respect to $n_{r,j} = (n_{r,j}, n_{r,j}, \ldots, n_{r,j})$, $\forall l' \in \zeta' = \{1, 2, \ldots, m\}, j \in J$ is:

$$
\nabla_{n_{r,j}} G_m = \left( \sum_{l=1}^{\infty} \sum_{k=K_l}^{\infty} n_{r,j}^k \delta_{l,j}^k \right), \quad \nabla_{n_{r,j}} G_m = \left( \sum_{l=1}^{\infty} \sum_{k=K_l}^{\infty} n_{r,j}^k \delta_{l,j}^k \right), \quad \nabla_{n_{r,j}} G_m = \left( \sum_{l=1}^{\infty} \sum_{k=K_l}^{\infty} n_{r,j}^k \delta_{l,j}^k \right), \quad \nabla_{n_{r,j}} G_m = \left( \sum_{l=1}^{\infty} \sum_{k=K_l}^{\infty} n_{r,j}^k \delta_{l,j}^k \right), \quad (A.2)
$$

And $F_{m,j}$ denotes the column vector of $\nabla_{n_{r,j}} G_m, l' \in \zeta' = \{1, 2, \ldots, m\}$:

$$
F_{m,j} = \left( \nabla_{n_{r,j}} G_m, \nabla_{n_{r,j}} G_m, \ldots, \nabla_{n_{r,j}} G_m \right)^T, \quad \forall m \in \zeta, j \in J. (A.3)
$$

Let $H_{m,j}(s)$ denote the Jacobian matrix of $F_{m,j}$ with respect to $\mathbf{n}_s = (\mathbf{n}_{1,s}, \mathbf{n}_{2,s}, \ldots, \mathbf{n}_{m,s})$, $\forall s \in J$:

$$
H_{m,j}(s) = J_{F_{m,j}}(\mathbf{n}_s), \quad \forall m \in \zeta, s \in J. (A.4)
$$

Note that $H_{m,j}(s)$ is a zero matrix when $s \neq j$, and thus the Hessian matrix of $G_m$ with respect to $\mathbf{n} = (\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_m)$ can be written as:

$$
H_m = \begin{bmatrix}
H_{m,1} & O & \ldots & O \\
O & H_{m,2} & \ldots & O \\
\vdots & \vdots & \ddots & \vdots \\
O & O & \ldots & H_{m,\lvert J \rvert} \\
\end{bmatrix}, \quad \forall m \in \zeta. (A.5)
$$

Since the characteristic values $\lambda$ of $H_m$ satisfy

$$
|\lambda E - H_m| = \begin{vmatrix}
\lambda E - H_{m,1} & O & \ldots & O \\
O & \lambda E - H_{m,2} & \ldots & O \\
\vdots & \vdots & \ddots & \vdots \\
O & O & \ldots & \lambda E - H_{m,\lvert J \rvert} \\
\end{vmatrix} = 0, \quad \forall m \in \zeta, (A.6)
$$

where $E$ is the identity matrix, we have

$$
|\lambda E - H_m| = |\lambda E - H_{m,1}| \cdot |\lambda E - H_{m,2}| \cdot \ldots \cdot |\lambda E - H_{m,\lvert J \rvert}| = 0. (A.7)
$$

Note that for $H_{m,j}(j), \forall j \in J$, the first-order primary minors are non-negative, and $k$-order ($k \geq 2$) primary minors equal to 0. Thereby, $H_{m,j}(j)$ is positive semidefinite and its characteristic values are non-negative. Since, according to Eq. (A.7), the eigenvalues of $H_m$ are identical to those of $H_{m,j}(j), \forall j \in J, H_m$ is positive semidefinite, and $G_m$ is convex. Therefore, the second summation term of Eq. (10a) is convex. This completes the proof.

**Appendix B. Proof of The Convergence for The SAB Solution Method**

Let $\mathbf{\bar{o}}^p$ be the solution computed by the SAB solution method for inverse optimization at iteration $p$. As $\mathbf{o}^{p+1}$ is found at each iteration $p$ by solving the convex quadratic parameter optimization problem in the vicinity of $|\bar{o}^p - \mathbf{\bar{o}}^p| \leq \delta$, the sequence $\{\tilde{F}(\mathbf{n}(\mathbf{\bar{o}}^p))\}$ is monotonically decreasing, i.e., $\tilde{F}(\mathbf{n}(\mathbf{\bar{o}}^p)) > \tilde{F}(\mathbf{n}(\mathbf{\bar{o}}^{p+1})), \forall p$. As outlined in Section 3.2, the upper-level objective function $\tilde{F}(\mathbf{n}(\mathbf{\bar{o}}))$
is uniformly continuous in $\mathbf{\omega}$. Given that $\mathbf{\omega}$ is closed and bounded, then $\tilde{F}(n^*(\mathbf{\omega}))$ is also bounded. Therefore, the sequence $\{\tilde{F}(n^*(\mathbf{\omega}^p))\}$ is expected to converge to a local optimal solution.

For comparison with a globally convergent algorithm for bi-level programming of continuous network design problem, we have further refined and implemented the norm-relaxed method of feasible directions (NRMFD) algorithm, as introduced by Cawood and Kostreva (1994) and expanded upon by Wang et al. (2021), to solve the inverse optimization-based preference estimation problem. The NRMFD algorithm linearizes the implicit terms in the upper-level problem using first-order approximation derived through sensitivity analysis of the lower-level problem. In each iteration, it solves a convex quadratic optimization problem, formulated based on the linearized system of the upper-level problem to identify a feasible descent direction, while employing a trial-and-error method to determine the step size. The NRMFD algorithm is proved to converge for non-convex optimization problems and achieve global convergence if the lower-level problem has a unique solution (Wang et al., 2021).

According to Cawood and Kostreva (1994), a descent direction $d^p$ is determined through solving the convex quadratic optimization problem as follows:

$$\min_{\theta, d^p} \theta + \frac{\sigma}{2}(d^p)^T \cdot H \cdot d^p$$ \hspace{1cm} (B.1a)

\[ s.t. \quad \nabla_{\mathbf{\omega}} \tilde{F}(n^*(\mathbf{\omega}^p)) \cdot d^p \leq \theta \] \hspace{1cm} (B.1b)

$$-\mathbf{\omega}^p - d^p \leq \theta$$ \hspace{1cm} (B.1c)

where $\theta$ is a decision variable, $\sigma$ is a positive parameter and $H$ is a symmetric positive definite matrix. The term $\nabla_{\mathbf{\omega}} \tilde{F}(n^*(\mathbf{\omega}^p))$ represents the gradient of upper-level objective $\tilde{F}(n^*(\mathbf{\omega}))$ with respect to $\mathbf{\omega}$ at iteration $p$. Let $(\theta^p, d^p)$ denote the solution to problem (B.1). As detailed by Chen and Kostreva (1999), if $\mathbf{\omega}^p$ is not a Fritz John point of bi-level programming (14), then $\theta^p < 0$ and $d^p$ will be a feasible descent direction for programming (14) at $\mathbf{\omega}^p$. Furthermore, if the bi-level programming is well-defined, the convergent point of the NRMFD algorithm will be a KKT point, which provides a local optimal solution to programming (14). The detailed procedure of the refined NRMFD algorithm for programming (14) is as follows:

**Step 1 Initialization:** Set an arbitrary initial feasible parameter value $\mathbf{\omega}^0$, and set $p \leftarrow 0$;

**Step 2 Solve the lower-level problem:** Solve the lower-level rail corridor commuting equilibrium model with the given parameter value $\mathbf{\omega}^p$ to obtain $n^*(\mathbf{\omega}^p)$ and calculate the value of upper-level objective $\tilde{F}(n^*(\mathbf{\omega}^p))$;

**Step 3 Sensitivity analysis:** Compute the derivative of $n$ with respect to $\mathbf{\omega}^p$ according to the sensitivity analysis method;

**Step 4 Find the feasible descent direction:** Solve the quadratic optimization problem (B.1) to obtain the feasible descent direction $d^p$ for programming (14) at $\mathbf{\omega}^p$; If $\theta^p > 0$, then terminate and return $\mathbf{\omega}^p$ as $\mathbf{\omega}^*$ else go to Step 5;

**Step 5 Determine the step size:**

**Step 5.1 Initial step size:** Set $l \leftarrow 0$, and let the initial step size $r^i = s/d^p_{\text{max}}$, where $s$ is an appropriate number for first-order approximation and $d^p_{\text{max}}$ is the maximum value in $d^p$;

**Step 5.2 First-order approximation:** approximates $n^*(\mathbf{\omega})$ as follows:

$$n^*(\mathbf{\omega}) \approx n^*(\mathbf{\omega}^p) + \nabla_{\mathbf{\omega}} n^*(\mathbf{\omega}^p) \cdot r^i \cdot d^p$$ \hspace{1cm} (B.2)
Step 5.3 Tuning the step size: if \( \tilde{\omega}^p + r' \cdot d' \geq 0 \) and \( \tilde{F}(n^*(\tilde{\omega})) < \tilde{F}(n^*(\tilde{\omega}^p)) \), then let \( \tilde{\omega}^{p+1} = \tilde{\omega}^p + r' \cdot d' \), and go to Step 5.4; otherwise, set \( r' = \delta \cdot r' \), where \( \delta \in (0, 1) \), and go to Step 5.2.

Step 5.4 Solve the lower-level problem: Solve the lower-level rail corridor commuting equilibrium model with the given parameter value \( \tilde{\omega}^{p+1} \) to obtain \( n^*(\tilde{\omega}^{p+1}) \) and calculate the value of upper-level objective \( \tilde{F}(n^*(\tilde{\omega}^{p+1})) \).

Step 6 Termination: If \( \left| \tilde{F}(n^*(\tilde{\omega}^{p+1})) - \tilde{F}(n^*(\tilde{\omega}^p)) \right| / \tilde{F}(n^*(\tilde{\omega}^p)) \leq \delta' \), where \( \delta' \) is a pre-set error limit, then terminate and return \( \tilde{\omega}^{p+1} \) as \( \tilde{\omega}^* \); otherwise, set \( p \leftarrow p + 1 \), and go to Step 3.

In Step 5, the equilibrated departure rates \( n^*(\tilde{\omega}) \) is linearly approximated rather than solving it precisely for efficiency, and the range of variation for the preference parameters \( \tilde{\omega}^p \) is confined within an appropriate range of \( s \) to ensure high accuracy in approximating. If the step size is too large to effectively reduce the upper-level objective value, then the step size will be scaled down until the new point \( \tilde{\omega}^{p+1} \) becomes feasible and the value of upper-level objective \( \tilde{F}(n^*(\tilde{\omega}^{p+1})) \) is less than \( \tilde{F}(n^*(\tilde{\omega}^p)) \).

The same dataset described in Section 5.2.2 is further used to illustrate the solution process of NRMFD algorithm. The matrix \( H \) and parameter \( \sigma \) in the NRMFD have a significant impact on the convergence rate. Typically, \( H \) is set as an identity matrix and \( \sigma \) is set between 0.5 and 10 for a faster convergence (Wang et al., 2021). However, in our inverse optimization-based preference estimation problem, due to the different orders of magnitude in preference parameters, using an identity matrix for \( H \) may lead to divergence in the NRMFD algorithm. Through various parameter trials, we found a faster convergence rate is achieved when \( H \) is set as \( \text{diag}(10^9, 10^9, 10^9, 10^{10}) \) and \( \sigma \) as 4. The values for \( s \), \( \delta' \) and \( \delta' \) are set as 0.0002, 0.618 (the golden ratio) and \( 10^{-6} \), respectively.

![Figure B.1: Convergence performance of NRMFD algorithm.](image)

The convergence performance shown in Figure B.1 indicates that the SAB solution method can converge to a solution point that is almost identical to that obtained by the refined NRMFD algorithm. However, the NRMFD exhibits a slower rate of convergence in the subsequent iterations when fine-tuning the solution, suggesting a need for integration with more efficient approaches, such as the Euler-based approximation for adaptively generating the feasible step size (Wang et al., 2022), to enhance the convergence.
Appendix C. Proof that \((\overline{n}^q - n^q)\) Is A Feasible Descent Direction for The Optimization of Eqs. (13), (10b) and (10c)

Since \(\overline{n}^q\) is a feasible solution to the mathematical programming of Eqs. (13), (10b) and (10c), if the following inequality holds

\[
\nabla_n \tilde{Z}(n, \tilde{\omega})\big|_{n=\overline{n}^q} \cdot (\overline{n}^q - n^q) < 0 ,
\]

where \(\nabla_n \tilde{Z}(n, \tilde{\omega})\big|_{n=\overline{n}^q}\) is the gradient of the objective function (Eqs. (13)) at \(n^q\), then, \((\overline{n}^q - n^q)\) is a feasible descent direction for the mathematical programming.

Note that

\[
\nabla_{n_{i,j}} \tilde{Z}(n, \tilde{\omega}) = \ln n_{i,j}^k + 1 + C_{i,j}(n, \tilde{\omega}) , \quad \forall i \in \zeta, k \in K, j \in J_{i,k} .
\]

With the conservation constraint of Eq. (10b), we have \(\sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} (\overline{n}^q_{i,j} - n_{i,j}^q) = 0\). Hence

\[
\nabla_n \tilde{Z}(n, \tilde{\omega})\big|_{n=\overline{n}^q} \cdot (\overline{n}^q - n^q) = \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} \nabla_{n_{i,j}} \tilde{Z}(n, \tilde{\omega})\big|_{n=\overline{n}^q} \cdot (\overline{n}^q_{i,j} - n_{i,j}^q)
\]

\[
= \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} \ln n_{i,j}^k (\overline{n}_{i,j}^k - n_{i,j}^q) + \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} C_{i,j}(n^q, \tilde{\omega}) (\overline{n}_{i,j}^q - n_{i,j}^q) .
\]

Note that \(\overline{n}^q\) obtained in Step 2 can be regarded as the solution to the mathematical programming:

\[
\min_n \tilde{Z}(n, \tilde{\omega}) = \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} n_{i,j}^k \ln n_{i,j}^k + \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} n_{i,j}^k C_{i,j} (n^q, \tilde{\omega}) ,
\]

\[s.t. \quad \sum_{j \in J_{i,k}} n_{i,j}^k = N_i , \quad \forall i \in \zeta, k \in K_i , \]

\[n_{i,j}^k \geq 0 , \quad \forall i \in \zeta, k \in K_i, j \in J_{i,k} .
\]

The first-order optimality conditions of the above convex programming are sufficient for a global optimum. Obviously, if \(n^q\) is not the optimal solution of Eqs. (13), (10b) and (10c), then

\[
\tilde{Z}(\overline{n}^q, \tilde{\omega}) - \tilde{Z}(n^q, \tilde{\omega}) < 0 ,
\]

i.e.,

\[
\sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} (\overline{n}_{i,j}^q \ln \overline{n}_{i,j}^q - n_{i,j}^q \ln n_{i,j}^q) + \sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} C_{i,j}(n^q, \tilde{\omega}) (\overline{n}_{i,j}^q - n_{i,j}^q) < 0 .
\]

Given the close proximity of the values of \(\sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} \overline{n}_{i,j}^q \ln \overline{n}_{i,j}^q\) and \(\sum_{i \in \zeta} \sum_{k \in K_i} \sum_{j \in J_{i,k}} \overline{m}_{i,j}^q \ln m_{i,j}^q\) (Huang and Lam, 1992), a comparison between Eq. (C.3) and Eq. (C.6) leads us to conclude that Eq. (C.3) holds. This completes the proof.

Appendix D. Equilibrium Properties for A Single Origin and
### Destination Rail Transit System

In a simple rail transit system serving a single origin and destination, the headways of train services are assumed to be very small to facilitate the analytical analysis of equilibrium properties, allowing the trains’ departure time to be treated as a continuous variable. In accordance with the generalized rail commuting cost of many-to-one corridor system in Eq. (4), the generalized commuting cost for commuters who depart at instant \( t \) can be formulated as:

\[
C(t) = \alpha T + \beta \max\left(t^* - t - T, 0\right) + \gamma \max\left(t + T - t^*, 0\right) + g_0n(t)T, \quad \forall t \in [t_B, t_E],
\]

where the travel time \( T \) is assumed to be in-vehicle time between a single origin and destination and can be seen as a constant. \([t_B, t_E]\) is the time window of morning peak period, where \( t_B \) is the start time and \( t_E \) is the end time.

When the equilibrium state is reached, commuters experience constant travel costs regardless of their departure time, i.e., \( d\overline{C}(t)/dt(t) = 0, \quad \forall t \in [t_B, t_E] \), where \( \overline{C}(t) \) denotes the equilibrium generalized commuting cost. The derivative of the equilibrium departure rate \( \overline{n}(t) \) is derived as follows, revealing that the maximum departure rate occurs at \( t^* \):

\[
\overline{n}(t) = \begin{cases} 
\frac{\beta}{g_0T}, & t_B \leq t \leq t^* \\
\frac{\gamma}{g_0T}, & t^* \leq t \leq t_E
\end{cases}
\]

At equilibrium, no commuters will choose to depart at \( t_B \) or \( t_E \), because if they were to depart at these times, they could reduce their commuting costs by either advancing their departure before \( t_B \) or delaying it after \( t_E \). Then we have:

\[
\overline{C}(t_B) = \alpha T(t) + \beta\left(t^* - t_B - T(t)\right), \quad (D.3a)
\]
\[
\overline{C}(t_E) = \alpha T(t) + \gamma\left(t_E + T(t) - t^*\right), \quad (D.3b)
\]
\[
\overline{C}(t) = \overline{C}(t_B) = \overline{C}(t_E). \quad (D.3c)
\]

Combining Eqs. (C.3a) to (C.3c), we can derive the equilibrium departure rate \( \overline{n}(t) \) as follows:

\[
\overline{n}(t) = \begin{cases} 
\frac{\beta(t - t_B)}{g_0T}, & t_B \leq t \leq t^* \\
\frac{\gamma(t - t_E)}{g_0T}, & t^* \leq t \leq t_E
\end{cases}
\]

(D.4)

The equilibrium departure rate distribution is therefore the collective outcomes of commuters’ trade-offs between schedule delays and in-vehicle crowding. The equilibrium departure rate pattern for a single origin and destination rail transit system, illustrated in Figure D.1, is also reflected in the observed departure rate counts, thereby exhibiting a similar endogenous pattern.
As the total commuting demand during \([t_B, t_E]\) is determinate, we have:

\[
\int_{t_a}^{t_e} \bar{n}(t) \, dt = N.
\]  

(D.5)

By combining Eqs. (D.5), (D.4) and (D.3c), we can further obtain the start and end times of the morning peak period at equilibrium:

\[
t_B = -T + t^* - \sqrt{\frac{T \gamma (2N g_0 - T \beta - T \gamma)}{\beta (\beta + \gamma)}},
\]

(D.6a)

\[
t_E = -T + t^* + \sqrt{\frac{T \beta (2N g_0 - T \beta - T \gamma)}{\gamma (\beta + \gamma)}}.
\]

(D.6b)

Given the interactions of departure time choices among upstream and downstream commuters along a rail transit corridor, the equilibrium analysis can be further extended to scenarios involving multiple origins and multiple commuter classes. By characterizing the interdependence between choice dynamics and preference measurements, the rail commuting equilibrium model essentially emerges as a system of simultaneous equations. Correspondingly, the iterative solution procedure for the equilibrium can be regarded as an evolutionary dynamic decision-making process that account for the lagged effects of travel costs and behavioral inertia, which ultimately converges to a specific equilibrium departure time choice pattern based on the given utility preferences. Therefore, through equilibrium-based inverse optimization, we can uncover commuters’ schedule preference and crowding perception by measuring the observed user optimality in their departure time choice patterns.

**Appendix E. Discrete Choice Estimation Using Two-Stage Method with Instrument Variables of Schedule Delays**

Assuming that commuters are homogeneous in travel time preferences and their desired arrival time corresponds to the peak value of arrival time feature as detailed in Section 5.2.1, from Figures 6 and 7 we can observe that during morning peak period, in-vehicle passenger flows generally increase as the desired arrival time approaches. This correlation suggests the potential for using time of schedule...
delays as instruments to address the endogeneity of in-vehicle crowding costs. Following the two-stage estimation method proposed by de Grange et al. (2024), which employs instrument variables similarly to control function method but uses them directly as explanatory variables instead of the endogenous variables, we attempt to address the endogeneity in crowding cost measurement using schedule delays as instrument variables. The perceived generalized travel cost experienced by commuter \( n \) taking train \( j \) from station \( O_i \) is given by:

\[
\tilde{C}_{n,i,j} = \alpha T(i,j) + \beta T_E(i,j) + \gamma T_L(i,j) + \epsilon_{i,j}, \quad \forall i \in \zeta^c, j \in J_i,
\]

(E.1)

where \( \epsilon_{i,j} \) is a non-observable random variable, the in-vehicle crowding measurement \( G(i,j) = \sum_{m=1}^{t} f(\hat{n}_{m,j})\tau_m \) is endogenous and we use time of schedule delays as instrument variables.

By assuming a linear relationship, \( G(i,j) \) can be written as follows:

\[
G(i,j) = \begin{cases} 
\alpha E(i,j) + a_{E,i} T_E(i,j) + \eta_{i,j}, & t_A(j) \leq t_i^* \\
\alpha L(i,j) + a_{L,i} T_L(i,j) + \eta_{i,j}, & t_A(j) > t_i^* 
\end{cases}
\]

(E.2)

where the coefficients of \( a \) are the parameters, \( \eta \) is random variable that is correlated with the error term of \( \epsilon_{i,j} \), and \( t_i^* \) is the desired arrival time.

Substituting Eq. (E.2) into Eq. (E.1), we obtain:

\[
\tilde{C}_{n,i,j} = \begin{cases} 
\alpha T(i,j) + \tilde{\beta} T_E(i,j) + b_{E,i} + \nu_{i,j}, & t_A(j) \leq t_i^* \\
\alpha T(i,j) + \tilde{\gamma} T_L(i,j) + b_{L,i} + \nu_{i,j}, & t_A(j) > t_i^* 
\end{cases}
\]

(E.3)

where \( \tilde{\beta} = \beta + g a_{E,i}, \tilde{\gamma} = \gamma + g a_{L,i}, b_{E,i} = g a_{E,i}^0, b_{L,i} = g a_{L,i}^0 \) and \( \nu_{i,j} = g \eta_{i,j} + \epsilon_{i,j} \). Assuming that the \( \nu_{i,j} \) are approximately independent and identically Gumbel-distributed (de Grange et al., 2024), the following multinomial logit formula can be derived:

\[
P_{n,i,j} = \frac{e^{-C_{n,i,j}}}{\sum_{s \in J_i} e^{-C_{n,s,j}}},
\]

(E.4)

where \( C_{n,i,j} = \alpha T(i,j) + \tilde{\beta} T_E(i,j) + \tilde{\gamma} T_L(i,j) + b_{i,j} \), and \( b_{i,j} = b_{E,i} \) if \( t_A(j) \leq t_i^* \), and \( b_{i,j} = b_{L,i} \) if \( t_A(j) > t_i^* \).

In the first stage, this method estimates \( \alpha, \tilde{\beta}, \tilde{\gamma} \) and \( b_{i,j} \) in Eq. (E.4) using the maximum likelihood estimation. As in discrete choice modeling, where the constant term in one of the utility functions of the alternatives should be taken as a reference, we set \( b_{L,i} = 0 \) and estimate \( b_{E,i} - b_{L,i} \).

In the second stage, the regression model of Eq. (E.2) is estimated to obtain the parameters of \( a \). Based on the estimators in the two stages, the schedule preference and in-vehicle crowding perception parameters of \( \tilde{\beta}, \tilde{\gamma} \) and \( g \) can be derived as follows:

\[
g = (b_{E,i} - b_{L,i}) / (a_{E,i}^0 - a_{L,i}^0), \quad \beta = \tilde{\beta} - ga_{E,i}, \quad \gamma = \tilde{\gamma} - ga_{L,i}.
\]

(E.5)

Taking homogenous commuters from Tu Qiao station as an example, Figure E.1 illustrates the correlation between in-vehicle crowding measurement and time of schedule delays. Since this potential relationship can only be identified among homogeneous commuters or specific commuter classes, this limitation makes it challenging to account for travel time preference heterogeneity, which is crucial for more precise estimations. Table E.1 summarizes the estimation results of the two stages. Substituting
these estimators into Eq. (E.5) yields the estimation results for $\beta$, $\gamma$ and $g$ as 0.001289, 0.001148 and 0.000389, respectively, with the sign bias of $g$ corrected. The standard deviations or variances of the estimation results can further be calculated using the delta method (Oehlert, 1992). However, the trip time coefficient of $\alpha$ exhibits a wrong sign with very low significance, indicating that train capacity constraints and commuters’ riding behavior may potentially induce endogeneity in trip time, as discussed in Section 4.2.

![Figure E.1: Correlation between in-vehicle crowding measurement and schedule delays.](image)

Table E.1: Estimation results of the two-stage method.

| Parameter | Stage 1 | | Parameter | Stage 2 | |
|-----------|---------|----------------|-----------|---------|
| $\alpha$  | -0.000138 | -0.43          | $a_{E,i}$  | -1.310*** | -5.85 |
| $\beta$   | 0.000779*** | 34.44          | $a_{E,i}^0$ | 8415.343*** | 17.63 |
| $\gamma$  | 0.000565*** | 23.69          | $a_{L,i}$  | -1.496*** | -12.60 |
| $b_{E,j} - b_{L,j}$ | 0.296397*** | 7.89          | $a_{L,i}^0$ | 7654.072*** | 20.74 |

Appendix F. Discrete Choice Estimation with Panel Data Dynamics in Travel Behavior

F.1. Classification of commuters with a complete commuting record

According to the disaggregate analysis method outlined in Section 4.1, the travel patterns of commuters with a complete commuting record are clustered to discern their travel time preferences. The classification numbers are determined based on the maximum of silhouette index in Figure F.1, resulting in 9, 10, 8 and 10 for commuters from stations of Tu Qiao, Liyuan, Guoyuan and Tongzhou Beiyuan, respectively. The habitual temporal travel patterns of each commuter class are shown in Figure F.2, and the results of commuter classification are summarized in Table F.1.
Figure F.1: Silhouette index value for K-means clustering approach.

Figure F.2: Arrival time features of multi-class commuters from multiple stations.

Table F.1: Summary of commuter classification.

<table>
<thead>
<tr>
<th>Origins</th>
<th>Class</th>
<th>$t_{i,k}$</th>
<th>$J_{i,k}$</th>
<th>Class</th>
<th>$t_{i,k}$</th>
<th>$J_{i,k}$</th>
<th>Class</th>
<th>$t_{i,k}$</th>
<th>$J_{i,k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>1</td>
<td>8:15</td>
<td>(10, 29)</td>
<td>4</td>
<td>9:15</td>
<td>(26, 46)</td>
<td>7</td>
<td>9:35</td>
<td>(20, 46)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8:25</td>
<td>(10, 29)</td>
<td>5</td>
<td>8:05</td>
<td>(7, 32)</td>
<td>8</td>
<td>8:55</td>
<td>(15, 38)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8:45</td>
<td>(15, 29)</td>
<td>6</td>
<td>9:05</td>
<td>(23, 35)</td>
<td>9</td>
<td>8:35</td>
<td>(23, 42)</td>
</tr>
<tr>
<td>$2$</td>
<td>1</td>
<td>9:25</td>
<td>(28, 46)</td>
<td>5</td>
<td>9:35</td>
<td>(28, 42)</td>
<td>9</td>
<td>8:35</td>
<td>(15, 38)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8:15</td>
<td>(7, 22)</td>
<td>6</td>
<td>9:05</td>
<td>(23, 38)</td>
<td>10</td>
<td>9:05</td>
<td>(23, 42)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8:55</td>
<td>(15, 35)</td>
<td>7</td>
<td>8:05</td>
<td>(4, 22)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>8:35</td>
<td>(4, 32)</td>
<td>8</td>
<td>7:55</td>
<td>(4, 24)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
F.2. Integrating the behavioral inertia attribute into the generalized travel cost for individuals

According to the sequence of departure time choices made by each smart card-holders, individuals’ behavioral inertia in their choices is captured as the proportion of times the train alternative has been chosen previously. For commuter \( n \), the proportion of times train \( j \) has been chosen on day \( t \) is defined as:

\[
PC_{n,j,t} = \frac{M_{n,j,t-1}}{M_{n,t-1}}, \quad \forall j \in J, t \in D,
\]

where \( M_{n,j,t-1} \) represents the number of times train \( j \) has been chosen by commuter \( n \) in the previous \( t-1 \) days, and \( M_{n,t-1} \) is the total number of travel times for commuter \( n \) in the previous \( t-1 \) days.

Considering the potential impact of behavioral inertia on commuters’ day-to-day dynamic departure time choices, where commuters may tend to stick with the previously chosen alternative unless another alternative offers substantially higher utility to justify a switch, the generalized travel cost in Eq. (31) can be further modified by integrating \( PC_{n,j,t} \):

\[
C^k_{n,i,j,t} = \alpha T_{i,t-1}(i,j) + \beta T^k_{E,j-1}(i,j) + \gamma T^k_{L,j-1}(i,j) + g \sum_{m} \hat{f}(\hat{n}_{m,j,t-1}) r_m + \zeta PC_{n,j,t},
\]

\[
\forall i \in \zeta, k \in K_j, j \in J_{i,k}, t \in D,
\]

where \( \zeta \) is the parameter representing commuters’ behavioral preference towards previously chosen alternatives.

Table F.2 provides the estimation results of integrating the behavioral inertia attribute into the generalized travel cost using multinomial logit and mixed logit, respectively. The estimated parameters for schedule preference and crowding perception exhibit positive signs, aligning with the rational behavioral assumption. The highly significant negative value of \( \zeta \) suggests that commuters prefer choosing the train services they had chosen before, indicating a strong behavioral inclination towards past choices.

### Table F.2: Estimation results of integrating the behavioral inertia attribute.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Multinomial logit</th>
<th>Mixed logit (Normal distr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>t-test</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.000076</td>
<td>0.13</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.001985***</td>
<td>50.36</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.001966***</td>
<td>48.55</td>
</tr>
<tr>
<td>( g )</td>
<td>0.000022*</td>
<td>1.75</td>
</tr>
</tbody>
</table>
F.3. Discrete choice estimation for commuters with homogenous travel time preferences

Commuters originating from the same station are assumed to share homogeneous travel time preferences. Specifically, commuters from Tu Qiao, Liyuan, Guoyuan and Tongzhou Beiyuan have identical desired arrival times of 8:45, 8:55, 8:45, and 8:45, respectively. The corresponding train choice sets are (8,46), (5,46), (5,46), and (2,46), respectively. The same panel dataset discussed in Section 5.1, comprising 6,810 trip records from 454 commuters, is employed for estimating Eq. (31) through the multinomial logit model and mixed logit model. The estimation results are presented in Table F.3.

Table F.3: Estimation results derived from panel data of homogeneous commuters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Multinomial logit</th>
<th>Mixed logit (Normal distr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>t-test</td>
<td>Mean</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.002498***</td>
<td>0.002499***</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.000520***</td>
<td>0.000520***</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.000461***</td>
<td>0.000461***</td>
</tr>
<tr>
<td>$g$</td>
<td>-0.000119***</td>
<td>-0.000119***</td>
</tr>
<tr>
<td>Init log-likelihood</td>
<td>-23808.35</td>
<td>-23808.35</td>
</tr>
<tr>
<td>Final log-likelihood</td>
<td>-22062.53</td>
<td>-22062.46</td>
</tr>
<tr>
<td>Rho square</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Rho bar square</td>
<td>0.07</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Without considering heterogeneity in travel time preferences, the parameter estimates of crowding perception preference remain susceptible to sign bias. The test for the IIA property of logit model also yields less favorable outcomes compared to multi-class commuters. Out of the 46 conducted tests, only 13 provide support for the null hypothesis of IIA at a 10% significance level. Therefore, by identifying and controlling for preference heterogeneity, clustering may contribute to mitigating certain types of endogeneity issues arising from unobserved individual differences.

References


Emerging Technologies, 125:102952.


